LONGTON LANE PRIMARY SCHOOL

***‘Believe and Achieve’***

|  |  |
| --- | --- |
| Amendments made since last review  Teachers have gone through the policy for their year group checking and crossing out any methods that are not taught in school. | |
| Policy agreed / reviewed 19th March 2024 | Next review due Spring 2026 |
|  | |
| Signed on behalf of the Governing Body | Signed by headteacher |

Progression in Calculation Policy



September 2017

*NB. Users should familiarise themselves with the introduction (pp 2-10) to this document before referring to individual year group guidance.*

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**Introduction**

At the centre of the mastery approach to the teaching of mathematics is the belief that all pupils have the potential to succeed. They should have access to the same curriculum content and, rather than being extended with new learning, they should deepen their conceptual understanding by tackling challenging and varied problems. Similarly, with calculation strategies, pupils must not simply rote learn procedures but demonstrate their understanding of these procedures through the use of concrete materials and pictorial representations. This document outlines the different calculation strategies that should be taught and used in Years 1 to 6, in line with the requirements of the 2014 Primary National Curriculum.

**Background**

The 2014 Primary National Curriculum for mathematics differs from its predecessor in many ways. Alongside the end of Key Stage year expectations, there are suggested goals for each year; there is also an emphasis on depth before breadth and a greater expectation of what pupils should achieve.

One of the key differences is the level of detail included, indicating what pupils should be learning and when. This is suggested content for each year group, but schools have been given autonomy to introduce content earlier or later, with the expectation that by the end of each key stage the required content has been covered.

For example, in Year 2, it is suggested that pupils should be able to ‘add and subtract one-digit and two-digit numbers to 20, including zero’ and a few years later in Year 5, they should be able to ‘add and subtract whole numbers with more than four digits, including using formal written methods

(columnar addition and subtraction)’.

In many ways, these specific objectives make it easier for teachers to plan a coherent approach to the development of pupils’ calculation skills, and the expectations of using formal methods is rightly coupled with the explicit requirement for pupils to use multiple representations, including concrete manipulatives and images or diagrams –a key component of the mastery approach.

**Purpose**

The purpose of this document is threefold. Firstly, in this introduction, it outlines the structures for calculations, which enable teachers to systematically plan problem contexts for calculations to ensure pupils are exposed to both standard and non-standard problems. Secondly, it makes teachers aware of the strategies that pupils are formally taught within each year group, which will support them to perform mental and written calculations. Finally, it supports teachers in identifying appropriate pictorial representations and concrete materials to help develop understanding.

The policy only details the strategies; teachers must plan opportunities for pupils to apply these, for example, when solving problems, or where opportunities emerge elsewhere in the curriculum.

**How to use the document**

For each of the four rules of number, different strategies are laid out, together with examples of what concrete materials can be used and how, along with suggested pictorial representations. Please note that the concrete and pictorial representation examples are not exhaustive, and teachers and pupils may well come up with alternatives. The purpose of using multiple representations is to give pupils a deep understanding of a mathematical concept and they should be able to work with and explain concrete, pictorial and abstract representations, and explain the links between different representations. Depth of understanding is achieved by moving between these representations. For example, if a child has started to use a pictorial representation, it does not mean that the concrete

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cannot be used alongside the pictorial. If a child is working in the abstract, depth can be evidenced by asking them to exemplify their abstract working using a concrete or pictorial representation and to explain what they have done using the correct mathematical vocabulary; language is, of course, one abstract representation but is given particular significance in the national curriculum.

**Mathematical language**

The 2014 National Curriculum is explicit in articulating the importance of pupils using the correct mathematical language as a central part of their learning. Indeed, in certain year groups, the non-statutory guidance highlights the requirement for pupils to extend their language around certain concepts.

“The quality and variety of language that pupils hear and speak are key factors in developing their mathematical vocabulary and presenting a mathematical justification, argument or proof.”

*2014 Maths Programme of Study*

|  |  |  |  |
| --- | --- | --- | --- |
| Suggested language structures accompany each strategy |  |  |  |
|  |  |  |
| outlined in this document. These build on one another | ones | units |  |
| systematically, which supports pupils in making links between | is equal to | equals / makes |  |
| and across strategies as they progress through primary school. | zero | oh (the letter O) |  |

New vocabulary should be introduced in a suitable context (for example, with relevant real objects, manipulatives, pictures or diagrams) and explained precisely. High expectations of the mathematical language used are essential, with teachers modelling accurate mathematical vocabulary and expecting pupils’ responses to include it ***infullsentences***.

**Presentation of calculations**

You will see that throughout this document, calculations are presented in a variety of ways. It is important for pupils’ mathematical understanding to experience and work with calculations and missing numbers in different positions relative to the = symbol. Examples used in classwork and

independent work should reflect this.

**Estimation**

Pupils are expected to use their developing number sense from Year 1 to make predictions about the answers to their calculations. As their range of mental strategies increases, these predictions and, later, estimates should become increasingly sophisticated and accurate. All teaching of calculation should emphasise the importance of making and using these estimates to check, first, the sense and, later, the accuracy of their calculations.

**Developing number sense**

Fluency in arithmetic is underpinned by a good sense of number and an ability to understand numbers as both a concept (e.g. 7 is the value assigned to a set of seven objects) and as something resulting from a process (three beads and four more beads gives seven beads altogether or 3 + 4 = 7). Understanding that a number can be partitioned in many ways (e.g. 7 = 4 + 3; 5 + 2 = 7; 1 + 6 = 7) is key to being able to use numbers flexibly in calculating strategies. The part-whole model and, later, bar models, are particularly useful for developing a relational understanding of number. Pupils who

are fluent in number bonds (initially within ten and then within twenty) will be able to use the ‘Make ten’ strategy efficiently, enabling them to move away from laborious and unreliable counting strategies, such as ‘counting all’ and ‘counting on’. Increasing fluency in inefficient strategies will allow pupils to develop flexible and interlinked approaches to addition and subtraction. At a later stage,

applying multiplication and division facts, rather than relying on skip-counting, will continue to develop flexibility with number.

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Derek Haylock (2014); Mathematics Explained for Primary Teachers, p.56

*“In a technological age, in which most calculations are done on machines, it surely cannot be disputed that knowing which calculation to do is more important than being able to do the calculation.”*

**Structures and contexts for calculations**

There are multiple contexts (the word problem or real-life situation, within which a calculation is required) for each mathematical operation (i.e. addition) and, as well as becoming fluent with efficient calculating strategies, pupils also need to become fluent in identifying which operations are required. If they are not regularly exposed to a range of different contexts, pupils will find it difficult to

apply their understanding of the four operations. For each operation, a range of contexts can be

identified as belonging to one of the conceptual ‘structures’ defined below.

The **structure** is distinct from both the **operation** required in a given problem and the **strategy** that may be used to solve the calculation. In order to develop good number sense and flexibility when calculating, children need to understand that many strategies (preferably the most efficient one for them!) can be used to solve a calculation, once the correct operation has been identified. There is often an implied link between the given structure of a problem context and a specific calculating strategy. Consider the following question: A chocolate bar company is giving out free samples of their chocolate on the street. They began the day with 256 bars and have given away 197. How many do they have

remaining? The reduction context implicitly suggests the action of ‘taking away’ and might lead to a

pupil, for example, counting back or using a formal algorithm to subtract 197 from 256 (seeing the

question as 256 –197 = ?). However, it is much easier to find the difference between 197 and 256 by adding on (seeing the question as 197 + ?= 256). Pupils with well-developed number sense and a clear understanding of the inverse relationship between addition and subtraction will be confident in manipulating numbers in this way.

Every effort is made to include multiple contexts for calculation in the Mathematics Mastery materials but, when teachers adapt the materials (which is absolutely encouraged), having an awareness of the different structures and being sure to include a range of appropriate contexts, will ensure that pupils continue to develop their understanding of each operation. The following list should not be considered to be exhaustive but defines the structures (and some suggested contexts) that are specifically included in the statutory objectives and the non-statutory guidance of the national curriculum. Specific structures and contexts are introduced in the Mathematics Mastery materials at the appropriate time, according to this guidance.

**Importance of knowns vs unknowns and using part-whole understanding**

One of the key strategies that pupils should use to identify the correct operation(s) to solve a given problem (in day-to-day life and in word problems) is to clarify the known and unknown quantities and identify the relationships between them. Owing to the inverse relationship between addition and

subtraction, it is better to consider them together as ‘additive reasoning’, since changing which information is unknown can lead to either addition or subtraction being more suitable to calculate a

solution for the same context. For the same reason, multiplication and division are referred to as

‘multiplicative reasoning’. Traditionally, approaches involving key vocabulary have been the main strategy used to identify suitable operations but owing to the shared underlying structures, key words

alone can be ambiguous and lead to misinterpretation (see for example the question below about

Samir and Lena, where the key word ‘less’ might be identified, but addition is required to solve the problem).

A more effective strategy is to encourage pupils to establish what they know about the relationship between the known and unknown values and if they represent a part or the whole in the problem, supported through the use of part-whole models and/or bar models. In the structures exemplified

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below, the knowns and unknowns have been highlighted. Where appropriate, the part-whole relationships have also been identified. Pupils should always be given opportunities to identify and discuss these, both when calculating and when problem-solving.

**Standard and non-standard contexts**

Using key vocabulary as a means of interpreting problems is only useful in what are in this document

defined as ‘standard’ contexts, i.e. those where the language is aligned with the operation used to solve the problem. Take the following example:

*First there were 12 people on the bus. Then three* ***more*** *people got on. How many people are on the bus now?*

Pupils would typically identify the word ‘more’ and assume from this that they need to add the values together, which in this case would be the correct action. However, in non-standard contexts,

identifying key vocabulary is unhelpful in identifying a suitable operation. Consider this question:

*First there were 12 people on the bus and then some more people got on at the school. Now there are 15 people on the bus. How many people got on at the school?*

Again the word ‘more’ would be identified, and a pupil may then erroneously add together 12 and 15.

It is therefore much more helpful to consider known and unknown values and the relations between them.

Overexposure to standard contexts and lack of exposure to non-standard contexts will mean pupils are more likely to rely on ‘key vocabulary’ strategies, as they see that this works in most of the cases they encounter. It is therefore important, when adapting lesson materials, that non-standards contexts are used systematically, alongside standard contexts.

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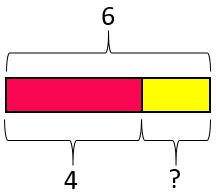
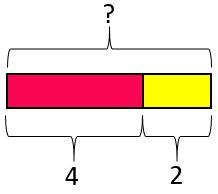
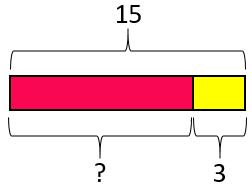
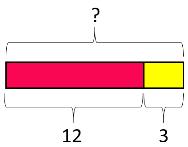
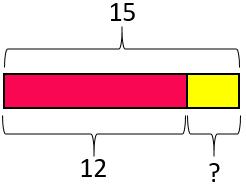
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Additive reasoning

**Change structures**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **augmentation (increasing)** | | **reduction (decreasing)** | |  |
| where an existing value has been added to | | where an existing value has been reduced | |  |
| **Standard** |  | **Standard** |  |  |
| First there were 12 people on the | | First Kieran had six plates in his | |  |
| bus. Then three more people got on. | | cupboard. Then he took two plates | |  |
| How many people are on the bus | | out to use for dinner. How many | |  |
| now? |  | plates are in the cupboard now? | |  |
| “I know both parts. My first part is twelve and my | | “I know the whole is six. I know one of the part that has | |  |
| second part is three. I don’t know the whole. I need to | | been taken away is two. I don’t know the other part. I | |  |
| add the parts of twelve and three to find the whole.” | | need to subtract the known part, two, from the whole, | |  |
| 12 + 3 = ? | | six, to find the remaining part.” | |  |
|  |  | 6 –2 = ? | 2 + ? = 6 |  |
| **Non-standard** |  |  |  |  |
| First there were 12 people on the | | **Non-standard** |  |  |
| bus and then some more people got | | First there were six plates in the | |  |
| on at the school. Now there are 15 | | cupboard. Then Kieran took some | |  |
| people on the bus. How many |  | out for dinner. There are now four | |  |
| people got on at the school? |  | plates left in the cupboard. How | |  |
| “I know my first part is twelve and I know the whole is | | many did Kieran take out? |  |  |
| 15. I don’t know the value of the second part. To find | | “I know the whole is six and the remaining part is four. I | |  |
| the second part, I could add on from 12 to make 15 or I | |  |
| don’t know the part that was taken away. To find the | |  |
| could subtract 12 from 15.” |  |  |
|  | part that was taken away I can add on from four to | |  |
| 12 + ? = 15 | 15 –12 = ? |  |
| make six or I could subtract four from six” | |  |
|  |  |  |
| **Non-standard** |  | 6 –? = 4 | 6 –4 = ? |  |
| First there were some people on the | |  |  |  |
| bus then it stopped to pick up three | | **Non-standard** |  |  |
| more passengers at the bank. |  | First there were some plates in the | |  |
| Altogether now there are 15 people | | cupboard. Then Kieran took two out | |  |
| on the bus. How many were people | | for dinner. Now there are four left. | |  |
| were on the bus before it stopped at | | How many plates were in the |  |  |
| the bank? |  | cupboard to start with? |  |  |
| “I know the value of the second part is three and that | | “I know the part that has been taken away is two and | |  |
| the whole is 15. I don’t know the value of the first part. | | the part that is left is four. I don’t know the whole. I can | |  |
| To find the first part, I could add on from three to make | | find the whole by adding the parts of four and two” | |  |
| 15 or I could subtract three from 15” | | ? –2 = 4 | 2 + 4 = ? |  |
| ? + 3 = 15 | 15 –3 = ? |  |  |  |
|  |  |  |  |  |



**Note**: the ‘first… then… now’ structure is used heavilyin KS1 to scaffold pupils’ understanding of change structures. Once pupils are confident with the structures, such linguistic scaffolding can be

removed, and question construction can be changed to expose pupils to a greater range of nuance in interpreting problems. For example, the second and third reduction problems could be reworded as follows:

Kieran took two plates out of his cupboard for dinner. There were four left. How many plates were in the cupboard to begin with?

There were six plates in the cupboard before Kieran took some out for dinner. If there were four plates left in the cupboard, how many did Kieran take out?

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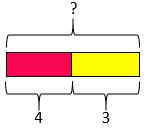
These present the same knowns and unknowns, and therefore the same bar models and resulting equations to solve the problems; however, the change in wording makes them more challenging to

pupils who have only worked with a ‘first… then… now’ structure so far.

**Part-whole structures**

**Combination (aggregation)/partitioning**

combining two or more discrete quantities/splitting one quantity into two or more sub-quantities



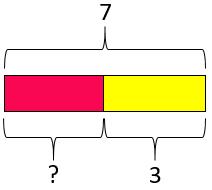
Hakan and Sally have made a stack of their favourite books. Four books belong to Hakan, three to Sally. How many books are in the stack altogether?

“I know both parts. One part is four and the other part is three. I don’t know the whole.

I need to add the parts of three and four to find the whole.”

4 + 3 = ? 3 + 4 = ?

*(Only one problem has been written for combination as, owing to the commutativity of addition, the only change in question wording would be to swap Hakan and Sally’s names. The resulting bar model and calculation would be identical.)*

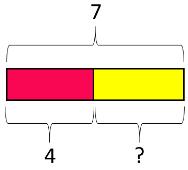


Sally and Hakan have made a stack of their favourite books. There are seven books altogether.

If three of them are Sally’s, how many belong to Hakan?

“I know the whole is seven and that one of the parts is three. I don’t know the other part. I need to

add on from three to make seven or subtract three from seven to find the other part.

3 + ? = 7 7 –3 = ?



Sally and Hakan have made a stack of their favourite books. There are seven books altogether.

If four of them are Hakan’s, how many belong to Sally?

“I know the whole is seven and that one of the parts is four. I don’t know the other part. I need

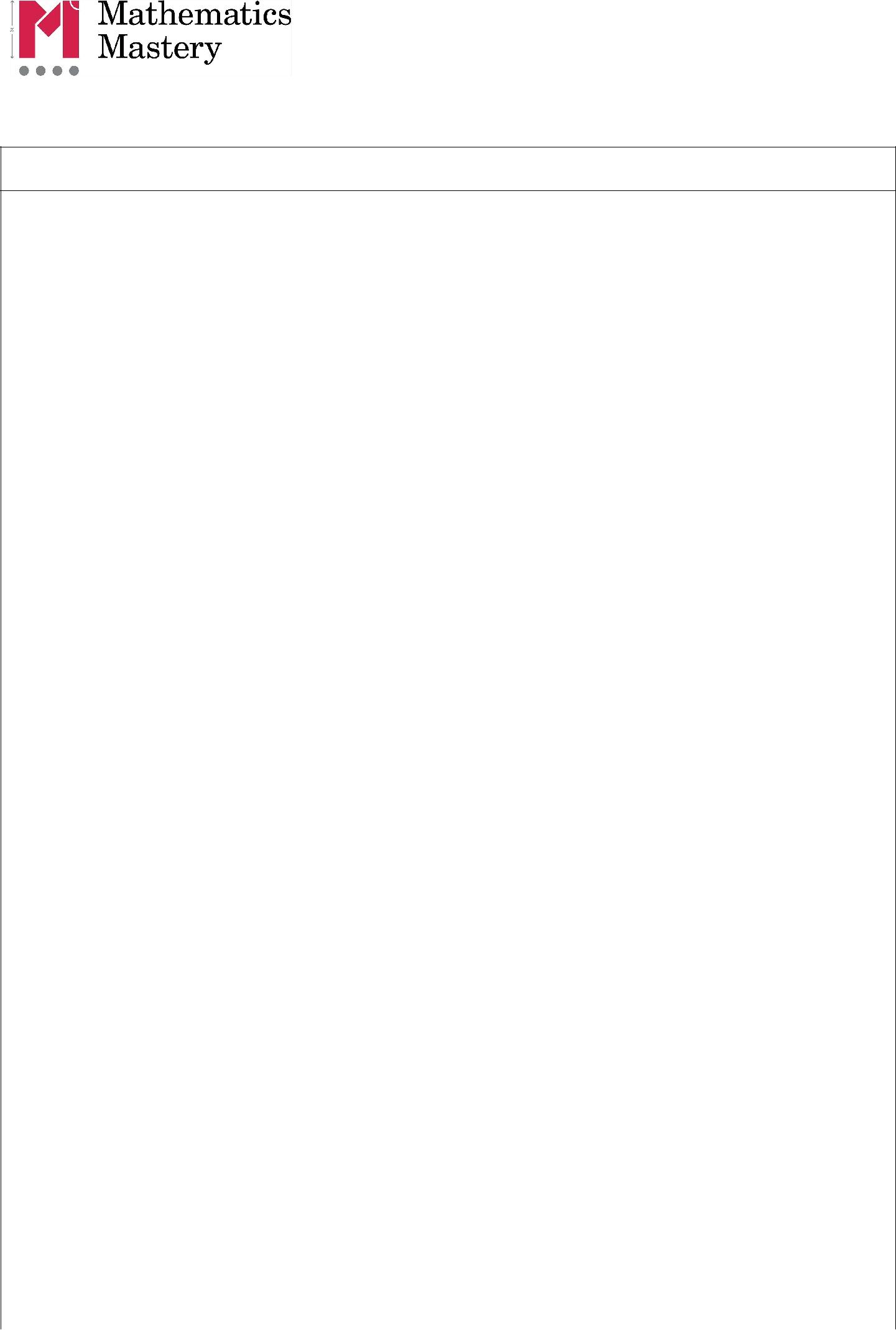
to add on from four to make seven or subtract four from seven to find the other part.”

4 + ? = 7 7 –4 = ?

**Note**: all part-whole contexts are considered tobe ‘standard’, as the language of part-whole isunambiguous.

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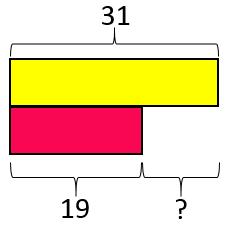


**Comparison structures**

Comparison structures involve a relationship between two quantities; their relationship is expressed as a difference. The structures vary by which of the values are known/unknown (the larger quantity, the smaller quantity and/or their difference). Part-whole language is not used here because the context contains not one single ‘whole’, but instead two separate quantities and it is the relationship between them being considered. Comparison bar models are therefore used to model these structures, which are known to be the most challenging for pupils to interpret.

**Smaller quantity and larger quantity are known (comparative difference)**

**Standard**

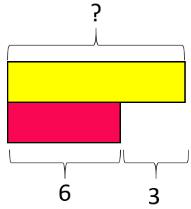


Navin has saved £19 from his pocket money. Sara has saved £31 from her pocket money. How much **more** has Sara saved than Navin? ***or*** How much **less** has Navin saved than Sara?

“I know one quantity is 19 and the other quantity is 31. I don’t know the difference. To find the

Difference I could add on from 19 to make 31 or I could subtract 19 from 31.”

19 + ? = 31 31 –19 = ?



**Smaller quantity and difference are known (comparative addition)**



**Standard**

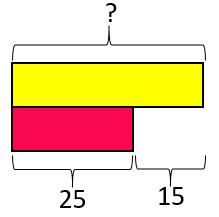
Ella has six marbles. Robin has three **more** than Ella. How many marbles does Robin have?

“I know the smaller quantity is six. I know the difference is three. I don’t know the larger

quantity. To find the larger quantity I need to add three to six.”

6 + 3 = ?

**Non-standard**



Samir and Lena are baking shortbread but Lena’s recipe uses 15g **less** butter than Samir’s. If

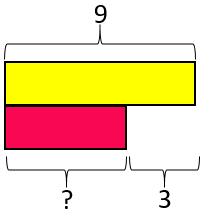
Lena needs to use 25g of butter, how much does Samir need?

“I know the smaller quantity is 25. I know the difference between the quantities is 15. I don’t

know the larger quantity. To find the larger quantity I need to add 15 to 25.”

? - 15 = 25 25 + 15 = ?

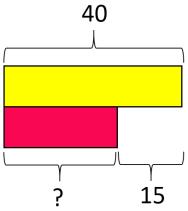
**Larger quantity and difference are known (comparative subtraction)**

**Non-standard**



Ella has some marbles. Robin has three **more** than Ella and he has nine marbles in total. How many marbles does Ella have?

“I know the larger quantity is nine. I know the difference between the quantities is three. I don’t know the smaller quantity. To find the smaller quantity I need to add on from three to make nine or subtract three from nine.”

? + 3 = 9 9 –3 = ?

**Standard**



Samir’s shortbread recipe uses 40g of butter. Lena’s recipe uses 15g **less** butter. How much

butter does Lena need?

“I know one quantity is 40. I know the difference between the quantities is 15. I don’t know the

smaller quantity but I know it is 15 less than 40. To find the smaller quantity, I need to subtract

15 from 40.”

40 –15 = ? ? + 15 = 40

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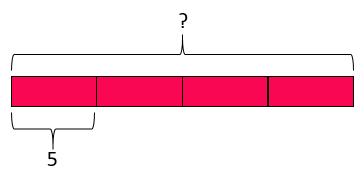
**Multiplicative reasoning**

**Repeated grouping structures**

**repeated addition**

groups (sets) of equal value are combined or repeatedly added

There are four packs of pencils. Each contains five pencils. How many pencils are there?



“I know there are four equal parts and that each part has a value of five. I don’t know the value of the whole. To find the whole, I need to multiply four and five.”

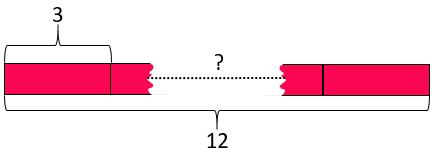
5 + 5 + 5 + 5 = ?

5 × 4 = ?

**repeated subtraction (grouping)**

groups (sets) of equal value are partitioned from the whole or repeatedly subtracted

There are 12 counters. If each child needs three counters to play the game, how many children can play?



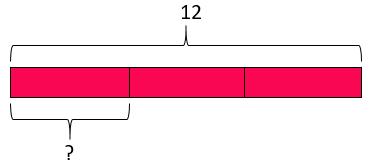
“I know the whole is twelve and that the value of each equal part is three. To find the number of equal parts, I need to know how many threes are in twelve.”

3 × ? = 12 12 ÷ 3 = ?

**sharing (into equal groups)**

the whole is shared into a known number (must be a positive integer) of equal groups (sets)

Share twelve counters equally between three children. How many counters does each child get?



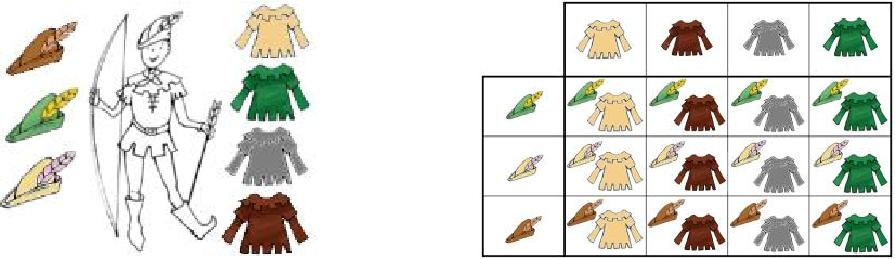
“I know the whole is twelve and the number of equal parts is three. I don’t know the value of each part. To find the value of each part, I need to know what goes into twelve three times.”

? × 3 = 12 12 ÷ 3 = ?

**Cartesian product of two measures**

**correspondence**

calculating the number of unique combinations that can be created from two (or more) sets



Robin has three different hats and four different tops. How many different outfits can he create, that combine one hat and one top?

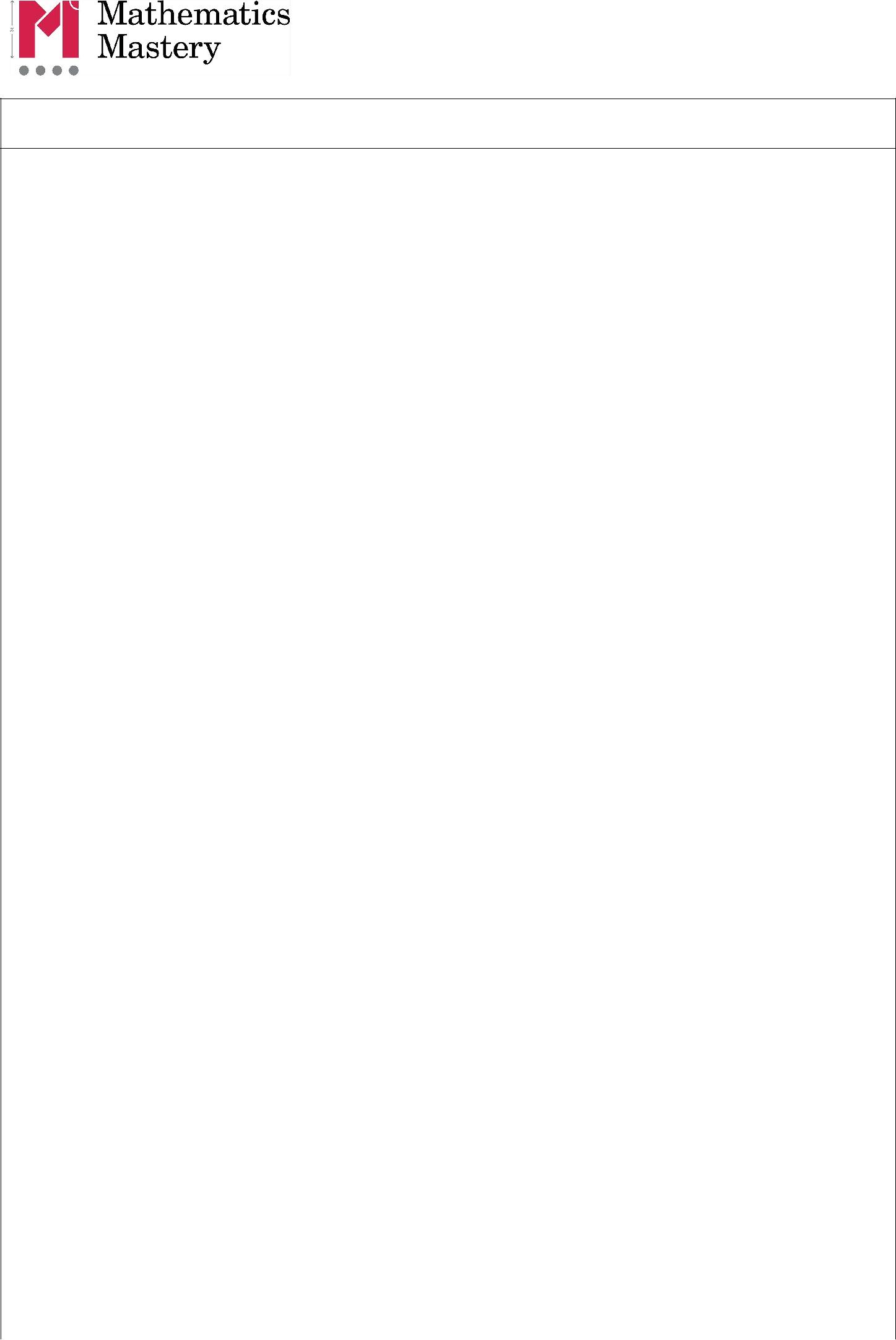


“I know how many hats there are and I know how many tops there are. I don’t know the different number of outfits that can be created. To find the number of outfits, I need to find how many different tops can be worn with each hat or how many different hats can be worn with each top.”

4 × 3 = ? 3 × 4 = ?

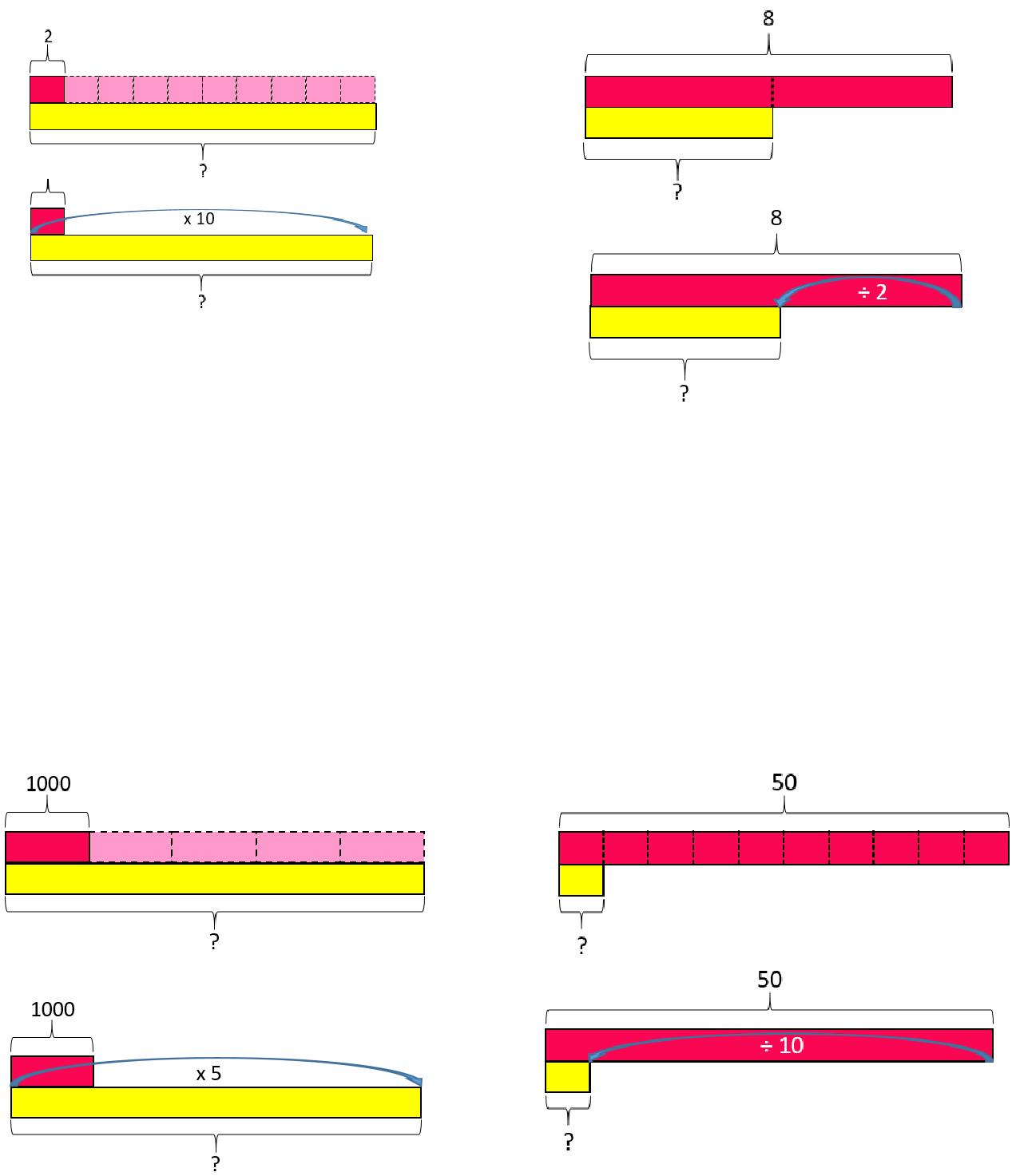
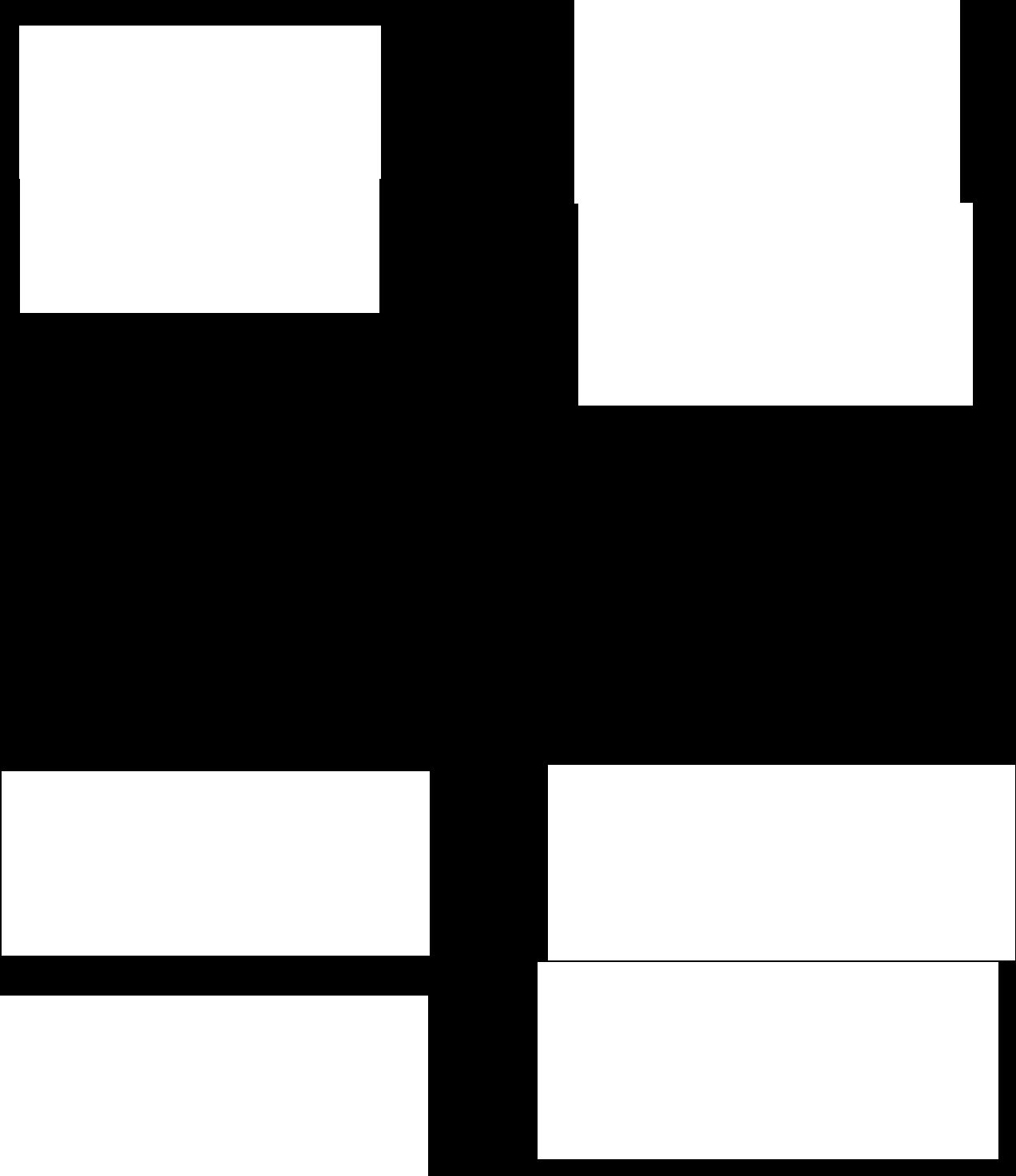
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9



**Scaling structures**

|  |  |  |  |
| --- | --- | --- | --- |
| **scaling up** | **scaling down** | |  |
| **(‘times greater / times as much’)** | **(‘times smaller/times less’)** | |  |
| the original value is increased by a given scale | the original value is reduced by a given scale factor | |  |
| factor | The house in my model village needs to be | |  |
|  |  |
| Rita receives £2 pocket money every week. Sim earns | half the height of the church. If the church is 8 cm tall, | |  |
| ten times as much money for her paper round. How | how tall does the house need to be? | |  |
| much money does Sim earn? |  |  |  |
| “I know one value is two and I know the second value is |  |  |  |
| ten times greater. I don’t know the second value. To |  |  |  |
| find the second value, I need to multiply two by ten.” |  |  |  |
| 2 × 10 = ? | “I know one value is eight and I know the second value is | |  |
|  | half as great. I don’t know the second value. To find the | |  |
|  | second value, I need to halve eight (or divide it by two).” | |  |
|  | Half of 8 is ? | 8 ÷ 2 = ? |  |
| **scaling up (‘times as many’)** | **scaling down (‘times fewer’)** | |  |
| the value of the original quantity is increased by a | the value of the original quantity is decreased by a | |  |
| given scale factor | given scale factor |  |  |
| The Albert Hall can hold five times as many people as | Anouska’s garden pond has ten times fewer frogs than | |  |
| the Festival Hall. If the Festival Hall holds 1000 people, | fish. If there are fifty fish, how many frogs are there? | |  |
| how many does the Albert Hall hold? |  |  |  |
| “I know one value is 1000 and I know the second value | “I know one value is 50 and I know the second value is ten | |  |
| is five times greater. I don’t know the second value. To | times less. I don’t know the second value. To find the | |  |
| find the second value, I need to multiply 1000 by five.” | second value, I need to divide fifty by ten.” | |  |
| 1000 × 5 = ? | 50 ÷ 10 = ? |  |  |



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10



Progression in calculations Year 1

**National curriculum objectives linked to addition and subtraction**

**These objectives are explicitly covered through the strategies outlined in this document:**

* Add and subtract one-digit and two-digit numbers to 100, including zero (N.B. Year 1 N.C. objective is to do this with numbers to 20).
* Add and subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones, a two-digit number and tens, 2 two-digit numbers; add 3 one-digit numbers (Year 2).
* Represent and use number bonds and related subtraction facts within 20.
* Given a number, identify 1 more and 1 less.
* Show that addition of two numbers can be done in any order (commutative) but subtraction of one number from another cannot (Year 2).
* Recognise the inverse relationship between addition and subtraction and use this to solve missing number problems (Year 2).

**The following objectives should be planned for lessons where new strategies are being introduced and developed:**

* Read, write and interpret mathematical statements involving addition (+), subtraction (−) and equal (=) signs.
* Solve one-step problems that involve addition and subtraction, using concrete objects and

pictorial representations, and missing number problems, such as 7 = ?−9.

* Solve problems with addition and subtraction:

1. Using concrete objects and pictorial representations, including those involving numbers, quantities and measures

o Applying their increasing knowledge of mental methods

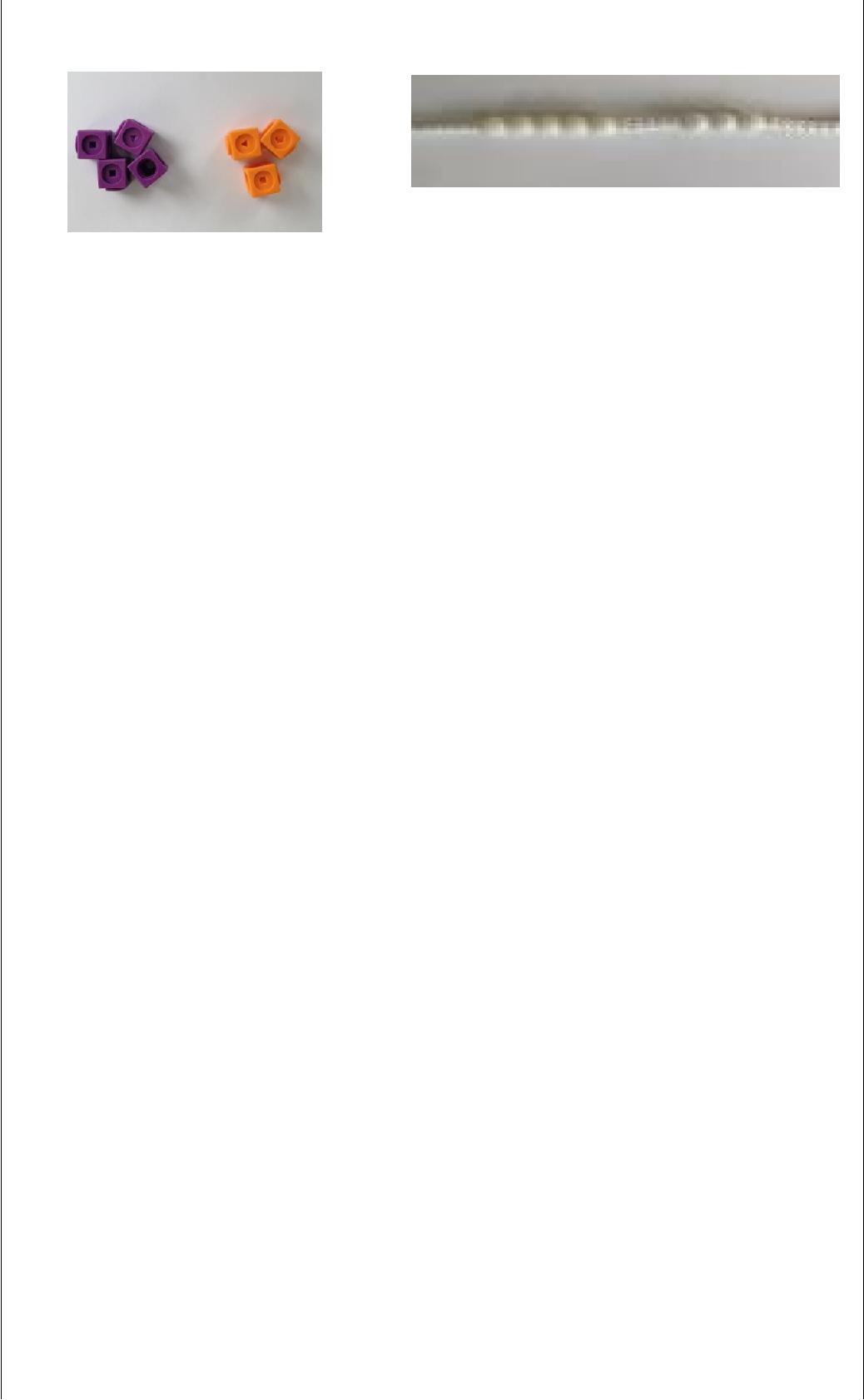
**Teachers should refer to the definitions and guidance on the**  [**structures for addition an**](#page5)**d**  [**subtraction**](#page5) **to provide a range of appropriate real-life contexts for calculations.**

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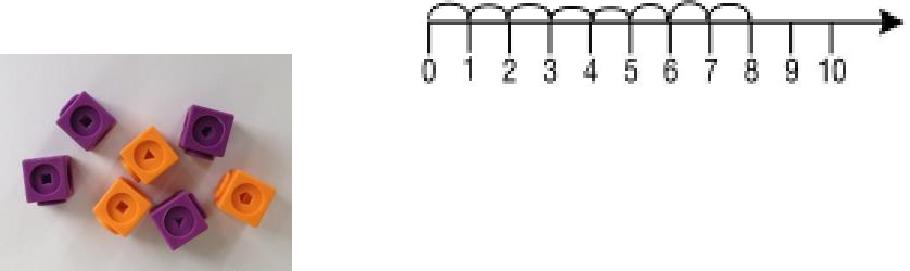
11



|  |  |
| --- | --- |
|  | **Y1 Addition** |
|  |  |
| **Strategy & guidance** | **CPA** |
|  |  |
| **Count all** | 3 + 4 = 7 |

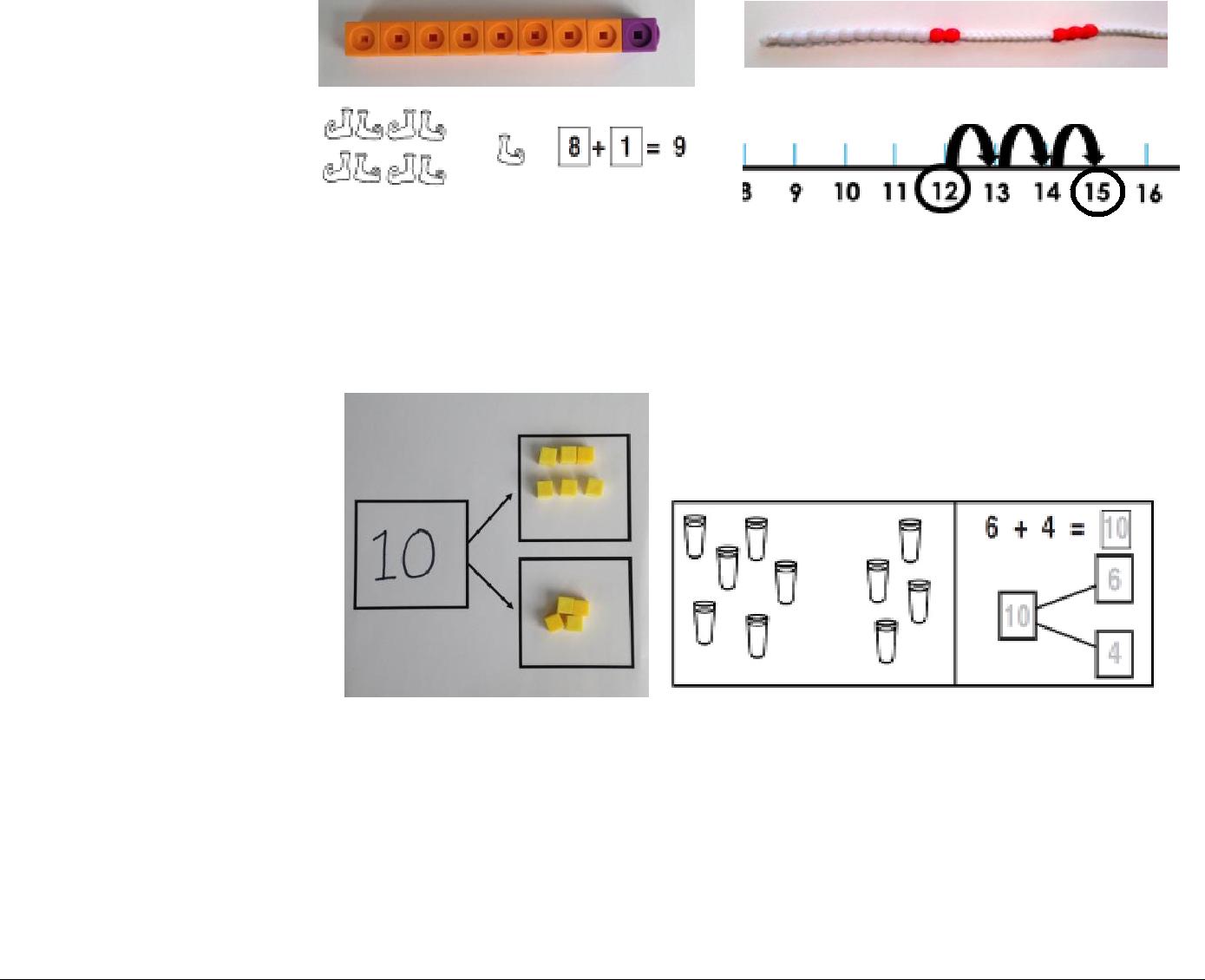
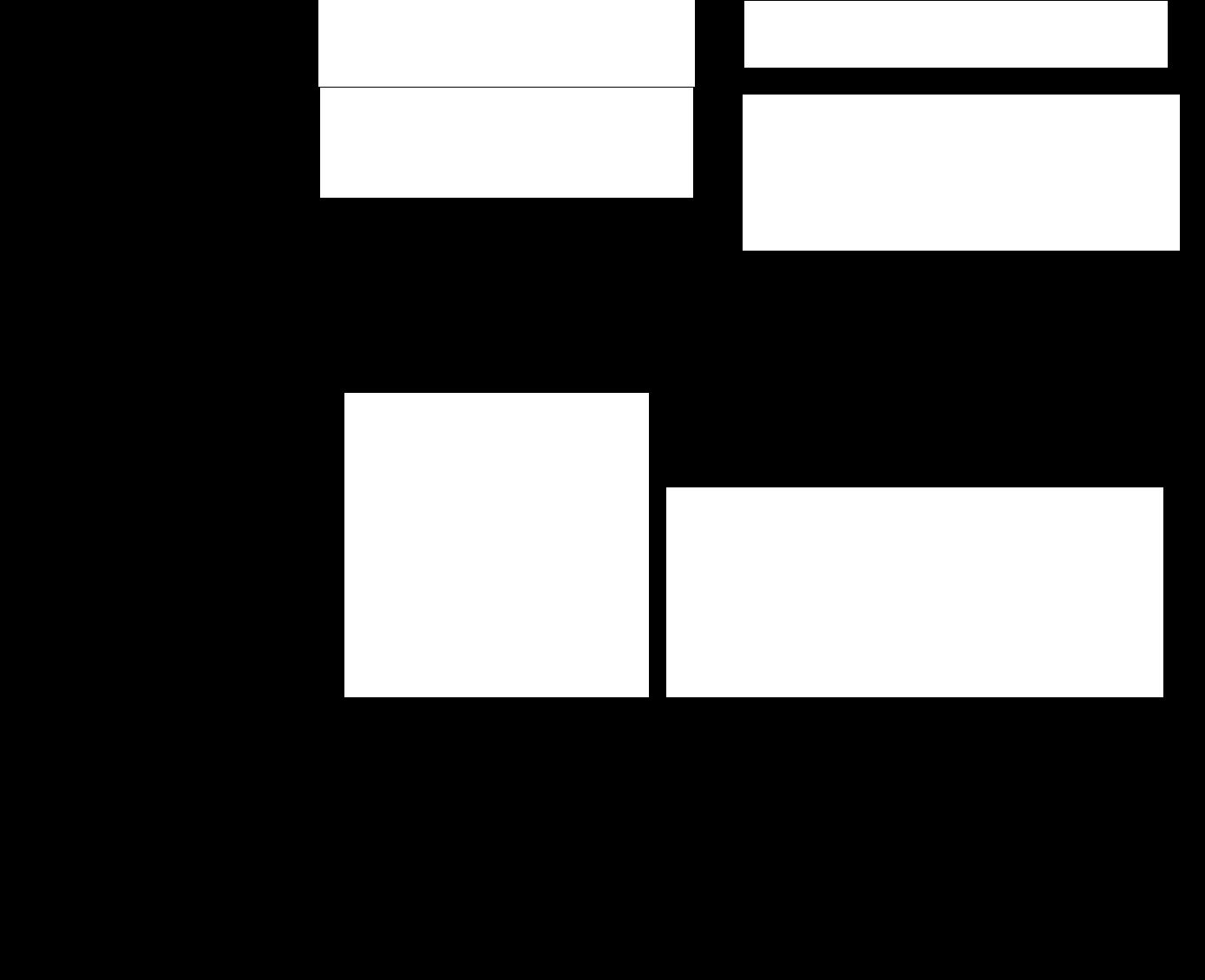


*Joining two groups and then recounting all objects using one-to-one correspondence*



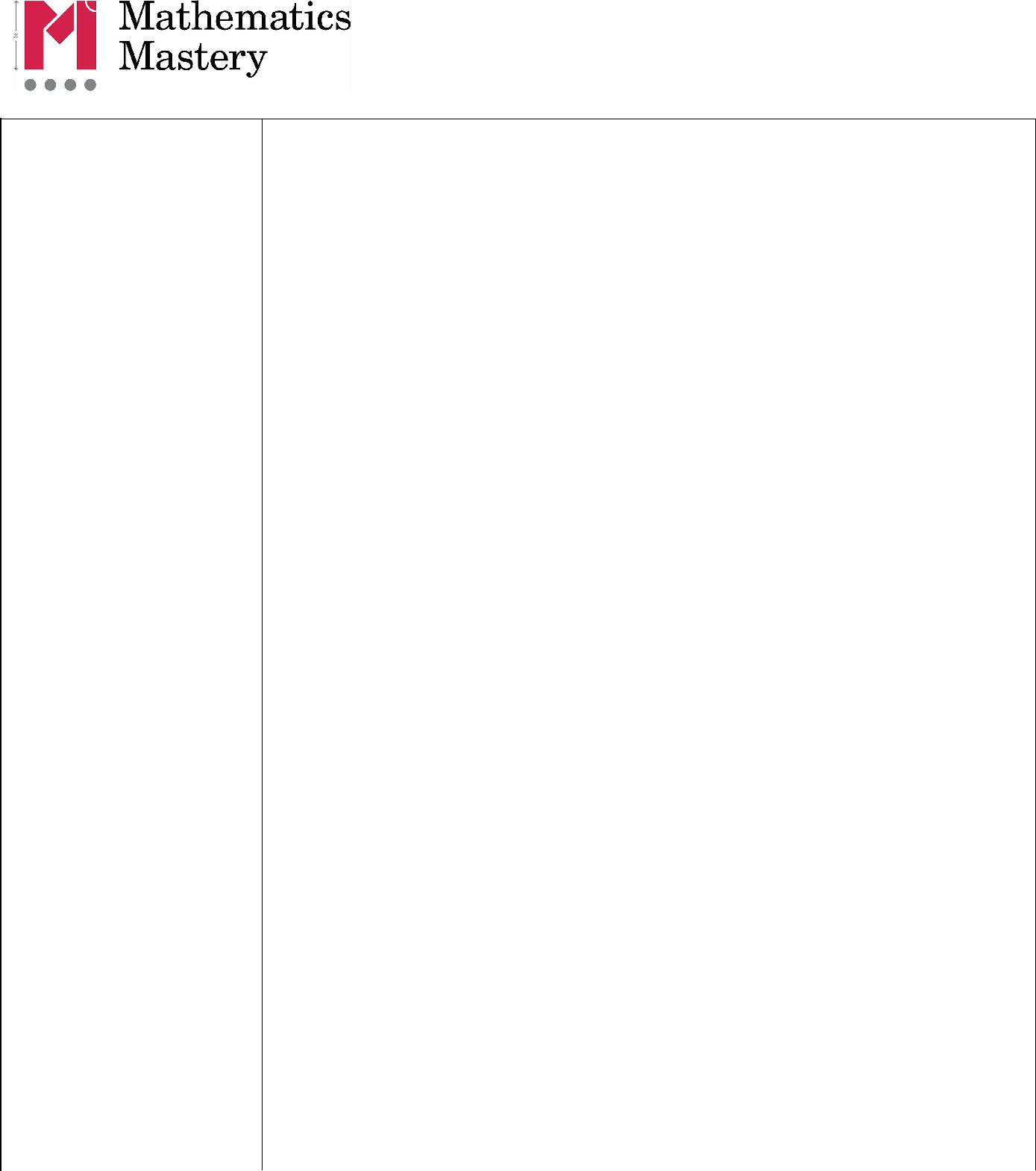
5 + 3 = 8

|  |  |  |  |
| --- | --- | --- | --- |
| **Counting on** | 8 + 1 = 9 | 15 = 12 + 3 |  |
| *As a strategy, this* |  |  |  |
| *should be limited to* |  |  |  |
| *adding small* |  |  |  |
| *quantities only (1, 2 or* |  |  |  |
| *3) with pupils* |  |  |  |
| *understanding that* |  |  |  |
| *counting on from the* |  |  |  |
| *greater number is* |  |  |  |
| *more efficient.* |  |  |  |
|  |  |  |  |
| **Part-part-whole** |  |  |  |
| *Teach both addition* |  |  |  |
| *and subtraction* |  |  |  |
| *alongside each other,* |  |  |  |
| *as pupils will use this* |  |  |  |
| *model to identify the* |  |  |  |
| *inverse relationship* |  |  |  |
| *between them.* |  |  |  |
| *This model begins to* |  | 10 = 6 + 4 |  |
| *develop the* |  |  |
|  |  |  |
| *understanding of the* |  | 10 −6 = 4 |  |
|  |  |  |
| *commutativity of* |  | 10 −4 = 6 |  |
| *addition, as pupils* |  | 10 = 4 + 6 |  |
| *become aware that the* |  |  |  |
| *parts will make the* |  |  |  |
| *whole in any order.* |  |  |  |

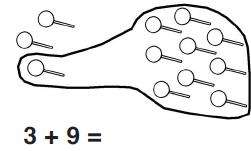


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12



**Regrouping ten ones to make ten**



*This is an essential skill that will support*

*column addition later*

*on.*

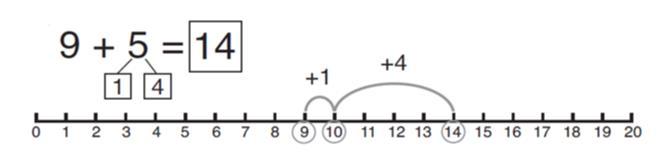


|  |  |  |
| --- | --- | --- |
| **‘Make ten’ strategy** | 6 + 5 = 11 | 4 + 9 = 13 |

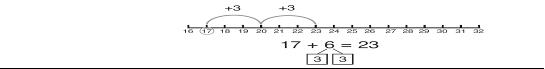


*Pupils should be encouraged to start at the greater number and partition the smaller number to make ten.*

*The colours of the beads on the bead string make it clear how many more need to be added to make ten.*



*Also, the empty spaces on the ten frame make it clear how many more are needed to make ten.*

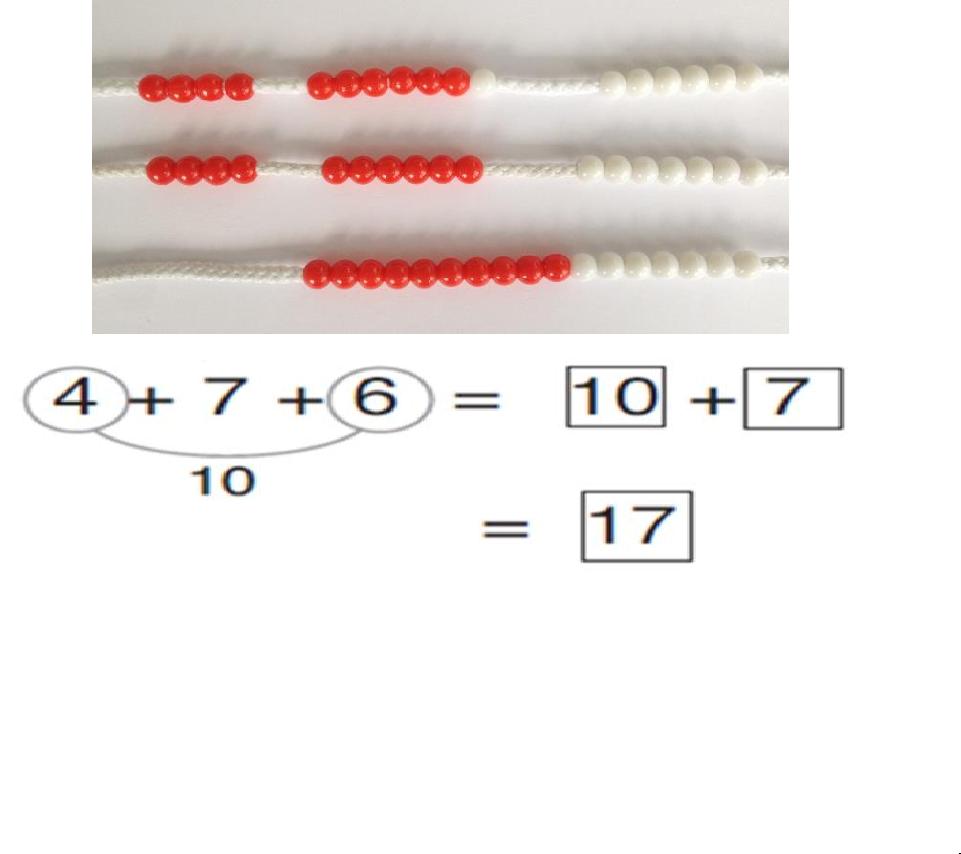
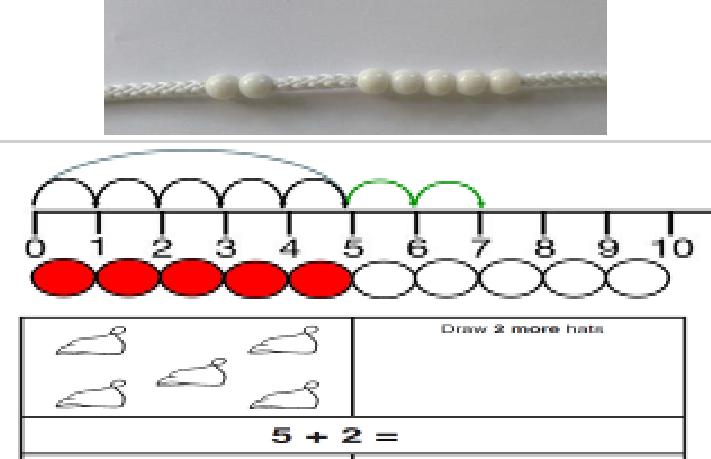


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13



|  |  |  |
| --- | --- | --- |
| **Adding 1, 2, 3 more** | 1 more than 5 | 5 + 1 = 6 |
| *Here the emphasis* |  |  |
| *should be on the* |  |  |
| *language rather than* |  |  |
| *the strategy. As pupils* |  |  |
| *are using the* |  |  |
| *beadstring, ensure that* | 2 more than 5 | 5 + 2 = 7 |
| *they are explaining* |  |  |
| *using language such* |  |  |
| *as;* |  |  |
| *‘1 more than 5 is equal* |  |  |
| *to 6.’* |  |  |
| *‘2 more than 5 is equal* |  |  |
| *to 7.’* |  |  |
| *‘8 is 3 more than 5’* |  |  |
| ***Over time, pupils*** |  |  |
| ***should be*** |  |  |
| ***encouraged to rely*** |  |  |
| ***more on their*** |  |  |
| ***number bonds*** |  |  |
| ***knowledge than on*** |  |  |
| ***counting strategies.*** |  |  |
|  |  |  |
| **Adding three single** |  |  |
| **digit numbers** |  |  |
| **(make ten first)** |  |  |
| *Pupils may need to try* |  |  |
| *different combinations* |  |  |
| *before they find the* |  |  |
| *two numbers that* |  |  |
| *make 10.* |  |  |
| *The first bead string* |  |  |
| *shows 4, 7 and 6. The* |  |  |
| *colours of the bead* |  |  |
| *string show that it* |  |  |
| *makes more than ten.* |  |  |
| *The second bead string* |  |  |
| *shows 4, 6 and then 7.* |  |  |
| *The final bead string* |  |  |
| *shows how they have* |  |  |
| *now been put together* |  |  |
| *to find the total.* |  |  |
|  |  |  |

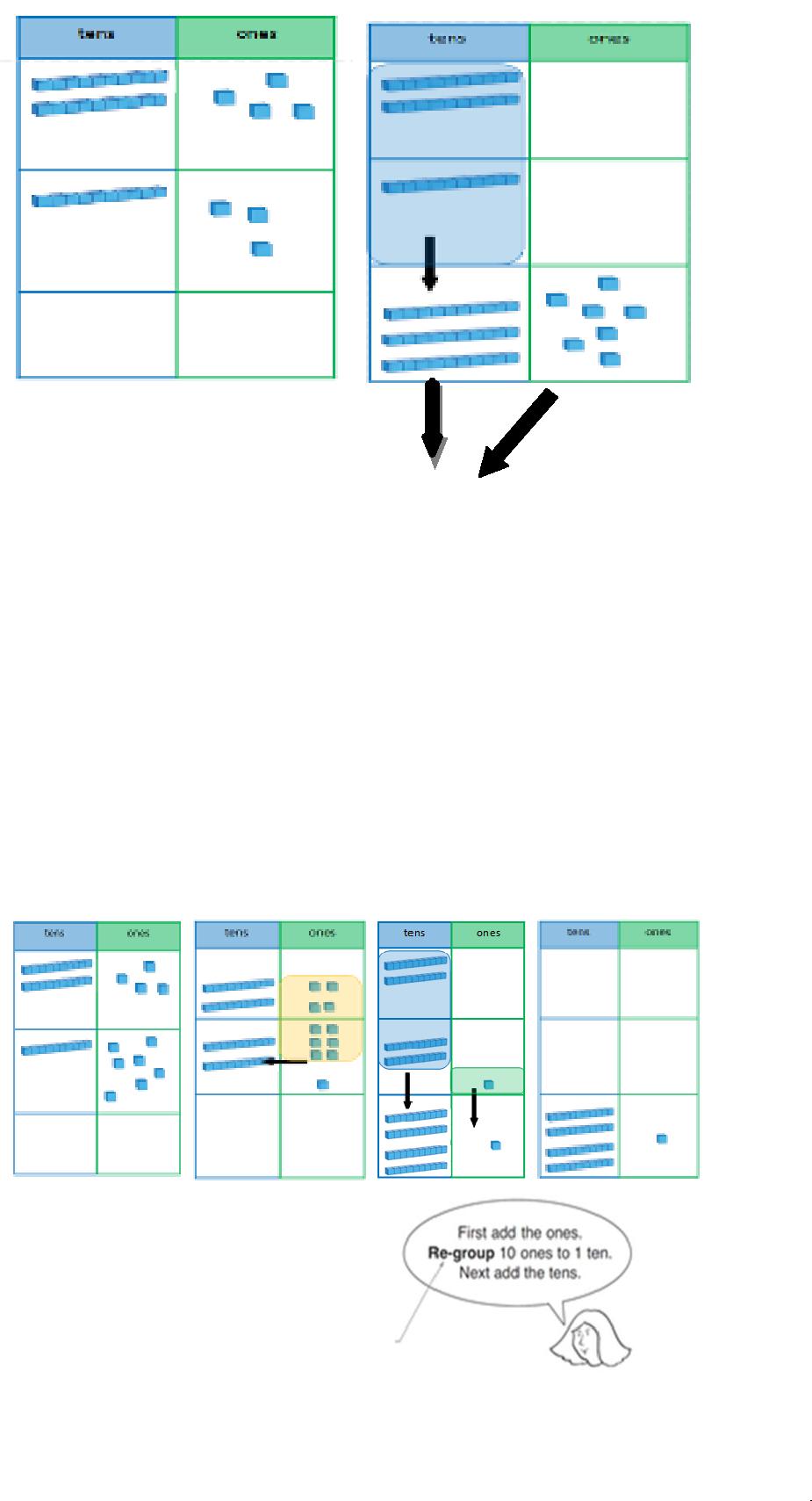


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14



|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Partitioning to add** | 24 + 13 = 37 | | | | |  |
| **(no regrouping)** |  |  |  |  |  |  |
| *Place value grids and* |  |  |  |  |  |  |
| *Dienes blocks could be* |  |  |  |  |  |  |
| *used as shown in the* |  |  |  |  |  |  |
| *diagram before* |  |  |  |  |  |  |
| *moving onto pictorial* |  |  |  |  |  |  |
| *representations. Dienes* |  |  |  |  |  |  |
| *blocks should always* |  |  |  |  |  |  |
| *be available, as the* |  |  |  |  |  |  |
| *main focus in Year 1 is* |  |  |  |  |  |  |
| *the concept of place* |  |  |  |  |  |  |
| *value rather than* |  |  |  |  |  |  |
| *mastering the* | 24 + 13 = 37 | | | | |  |
| *procedure.* |  |
| *When not regrouping,* |  |  |  |  |  |  |
| *partitioning is a* |  |  |  |  |  |  |
| *mental strategy and* |  |  |  |  |  |  |
| *does not need formal* |  |  |  |  |  |  |
| *recording in columns.* |  |  |  |  |  |  |
| *This representation* |  |  |  |  |  |  |
| *prepares them for* |  |  |  |  |  |  |
| *using column addition* |  |  |  |  |  |  |
| *with formal recording.* |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  | | | | |  |
| Column not taught in Year 1. |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  | Tens Ones | | | | |  |
|  |  |
|  | 2 4 | | |  |  |  |
|  |  |  |  |
| + 1 7 | | |  |  |  |
|  |  |  |  |
|  | 1 | |  |  |  |  |
|  |  |  |  |
|  |  |  |  |  |  |
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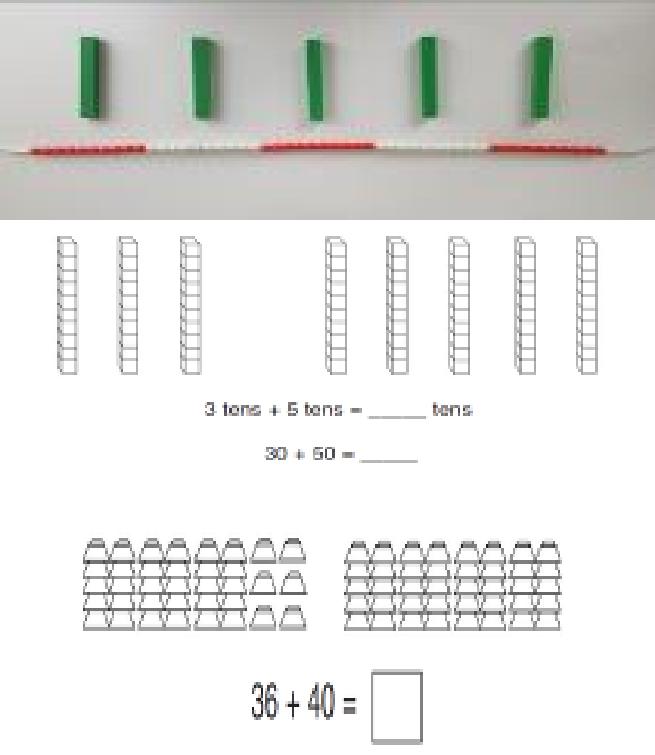


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15



|  |  |
| --- | --- |
| **Adding multiples of** | 50 = 30 + 20 |
| **ten** |  |
| *Using the vocabulary* |  |
| *of 1 ten, 2 tens, 3 tens* |  |
| *etc.* ***alongside*** *10, 20,* |  |
| *30 is important, as* |  |
| *pupils need to* |  |
| *understand that it is a* |  |
| ***ten*** *and not a one that* |  |
| *is being added and* |  |
| *they need to* |  |
| *Understand that a ‘2’* |  |
| *digit in the tens column* |  |
| *has a value of twenty.* |  |
| *It also emphasises the* |  |
| *link to known number* |  |
| *facts. E.g. ‘2+3 is* |  |
| *equal to 5. So 2 tens + 3* |  |
| *tens is equal to 5 tens.* |  |
|  |  |



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16



|  |  |  |
| --- | --- | --- |
|  | **Y1 Subtraction** |  |
|  |  |  |
| **Strategy & guidance** | **CPA** |  |
|  |  |  |
| **Taking away from** |  |  |
| **the ones** |  |  |
| *When this is first* |  |  |
| *introduced, the* |  |  |
| *concrete* |  |  |
| *representation should* |  |  |
| *be based upon the* |  |  |
| *diagram. Real objects* |  |  |
| *should be placed on* |  |  |
| *top of the images as* |  |  |
| *one-to-one* |  |  |
| *correspondence so* |  |  |
| *that pupils can take* |  |  |
| *them away,* |  |  |
| *progressing to* |  |  |
| *representing the group* |  |  |
| *of ten with a tens rod* |  |  |
| *and ones with ones* |  |  |
| *cubes.* |  |  |
|  |  |  |
| **Counting back** |  |  |
| *Subtracting 1, 2, or 3* | 16 –2 = 14 |  |
| *by counting back* |  |  |
| ***Pupils should be*** |  |  |
| ***encouraged to rely*** |  |  |
| ***on number bonds*** |  |  |
| ***knowledge as time*** |  |  |
| ***goes on, rather*** |  |  |
| ***than using*** |  |  |
| ***counting back as*** |  |  |
| ***their main*** | 4 = 6 − |  |
| ***strategy.*** |  |
|  |  |
|  |  |  |

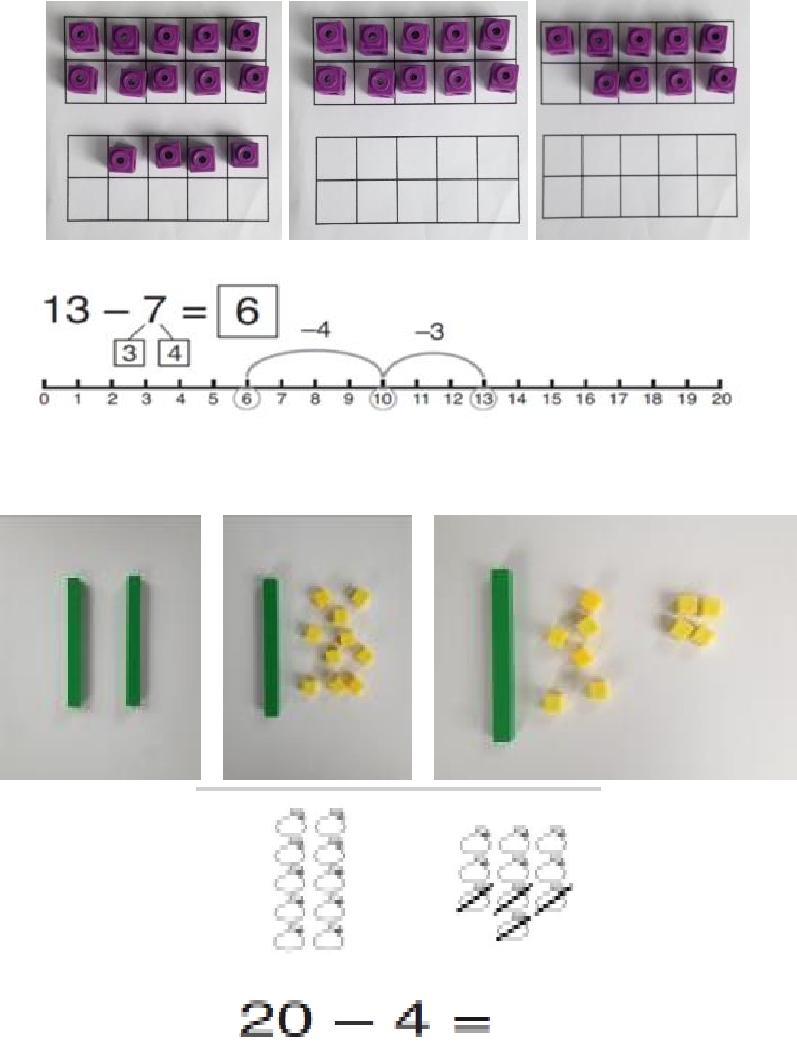
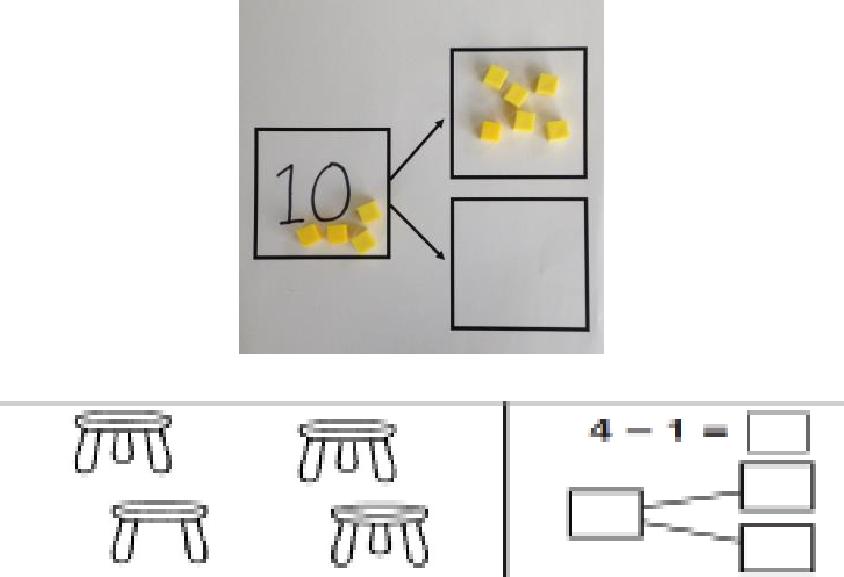


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17

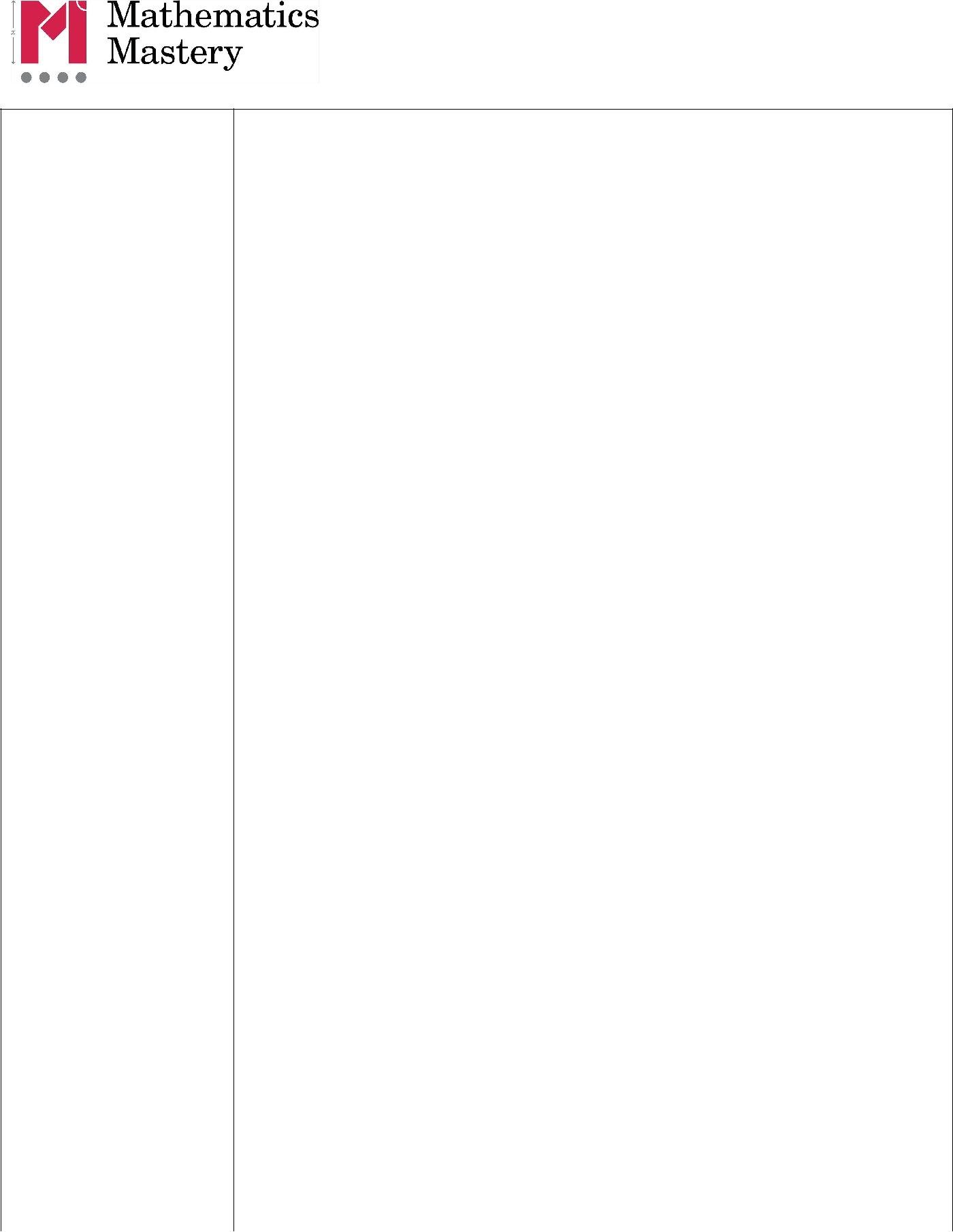


|  |  |
| --- | --- |
| **Part-part-whole** | 10 −6 = 4 |
| *Teach both addition* |  |
| *and subtraction* |  |
| *alongside each other,* |  |
| *as the pupils will use* |  |
| *this model to identify* |  |
| *the link between them.* |  |
| *Pupils start with ten* |  |
| *cubes placed on the* |  |
| *whole.* |  |
| *They then remove* |  |
| *what is being taken* |  |
| *away from the whole* |  |
| *and place it on one of* |  |
| *the parts.* |  |
| *The remaining cubes* |  |
| *are the other part and* |  |
| *also the answer. These* |  |
| *can be moved into the* |  |
| *second part space.* |  |
|  |  |
| **Make ten strategy** | 14 –5 = 9 |
| *To subtract a 1-digit* |  |
| *number from a 2-digit* |  |
| *number.* |  |
| *Pupils identify how* |  |
| *many need to be taken* |  |
| *away to make ten* |  |
| *first, partitioning the* |  |
| *number being* |  |
| *subtracted. Then they* |  |
| *take away the rest to* |  |
| *reach the answer.* |  |
|  |  |
| **Regroup a ten into** |  |
| **10 ones** |  |
| *After the initial* |  |
| *introduction, the* |  |
| *Dienes blocks should* |  |
| *be placed on a place* |  |
| *value chart to support* |  |
| *place value* |  |
| *understanding. This* |  |
| *will support pupils* |  |
| *when they later use* |  |
| *the column method*. |  |
|  |  |

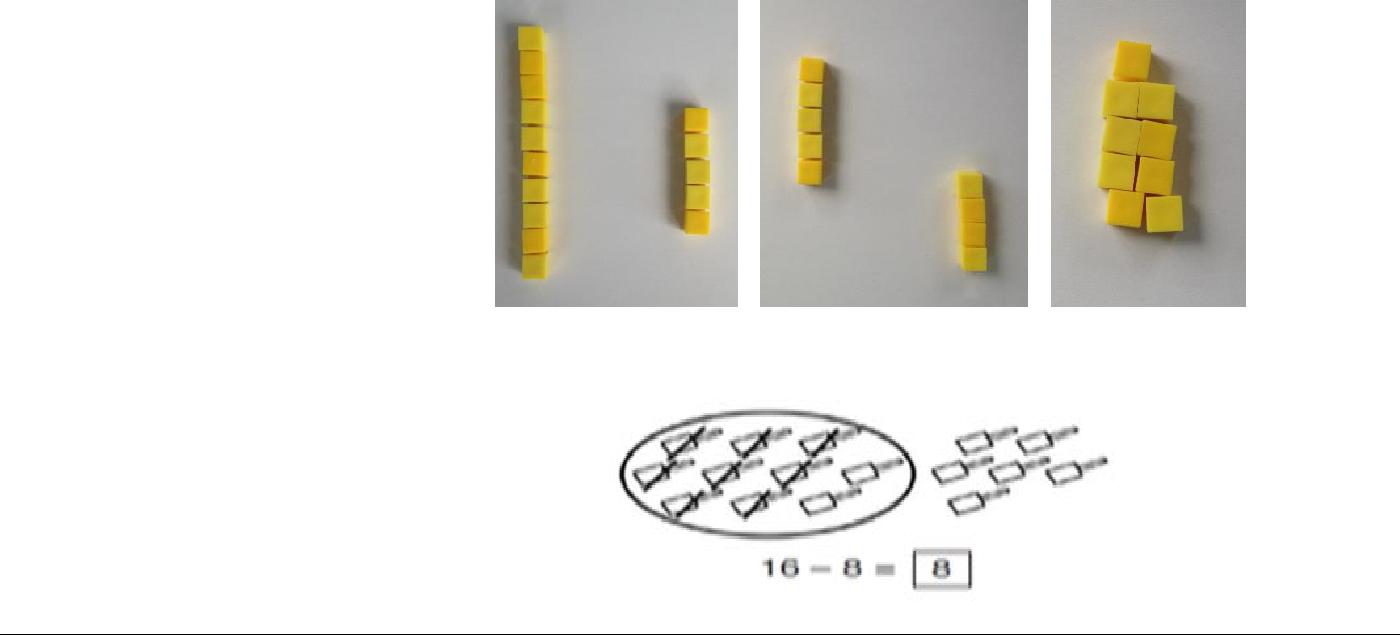
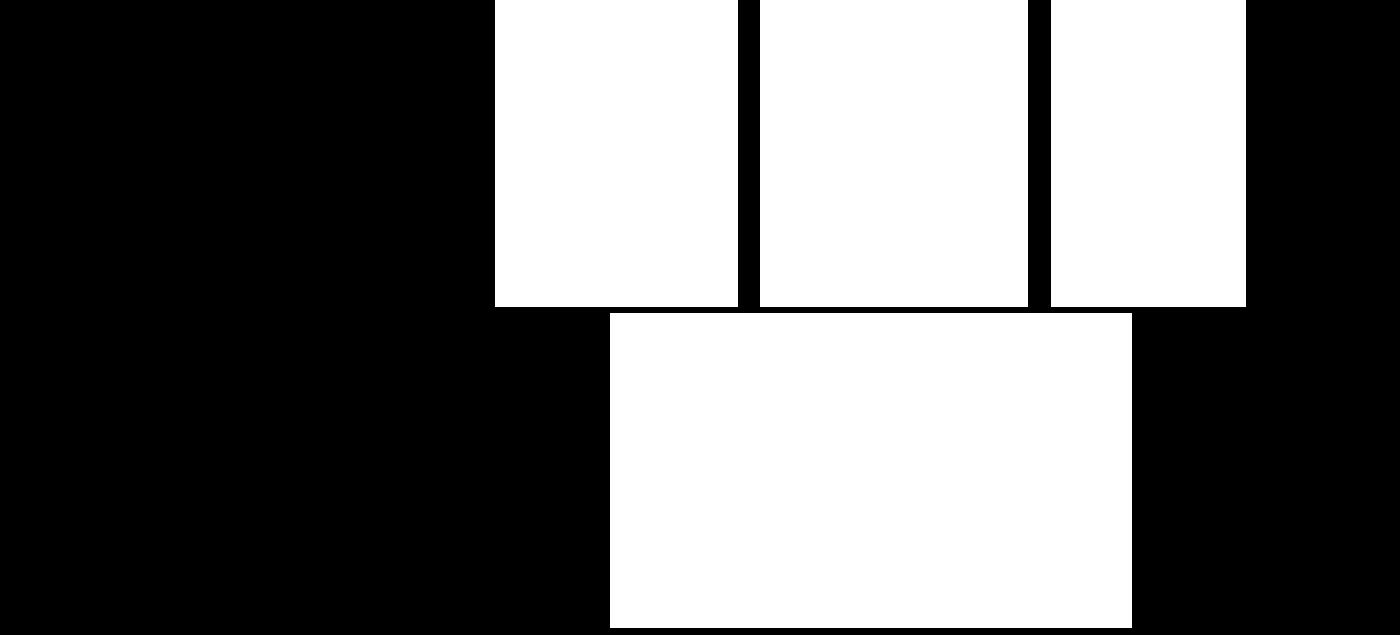


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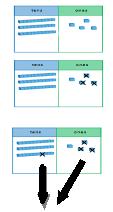
18



|  |  |
| --- | --- |
| **Taking away from** | 9 = 15−6 |
| **the tens** |  |
| *Pupils should identify* |  |
| *that they can also take* |  |
| *away from the tens* |  |
| *and get the same* |  |
| *answer.* |  |
| *This reinforces their* |  |
| *knowledge of number* |  |
| *bonds to 10 and* |  |
| *develops their* |  |
| *application of number* |  |
| *bonds for mental* |  |
| *strategies.* |  |



|  |  |
| --- | --- |
| **Partitioning to** | 34 −13 = 21 |
| **subtract without** |  |
| **regrouping** |  |
| *Dienes blocks on a* |  |
| *place value chart* |  |
| *(developing into using* |  |
| *images on the chart)* |  |
| *could be used, as when* |  |
| *adding 2-digit* |  |
| *numbers, reinforcing* |  |
| *the main concept of* |  |
| *place value for Year 1.* |  |
|  | 34 –13 = 21 |
| *When not regrouping,* |  |
| *partitioning is a* |  |
| *mental strategy and* |  |
| *does not need formal* |  |
| *recording in columns.* |  |
| *This representation* |  |
| *prepares them for* |  |
| *using column* |  |
| *subtraction with* |  |
| *formal recording.* |  |

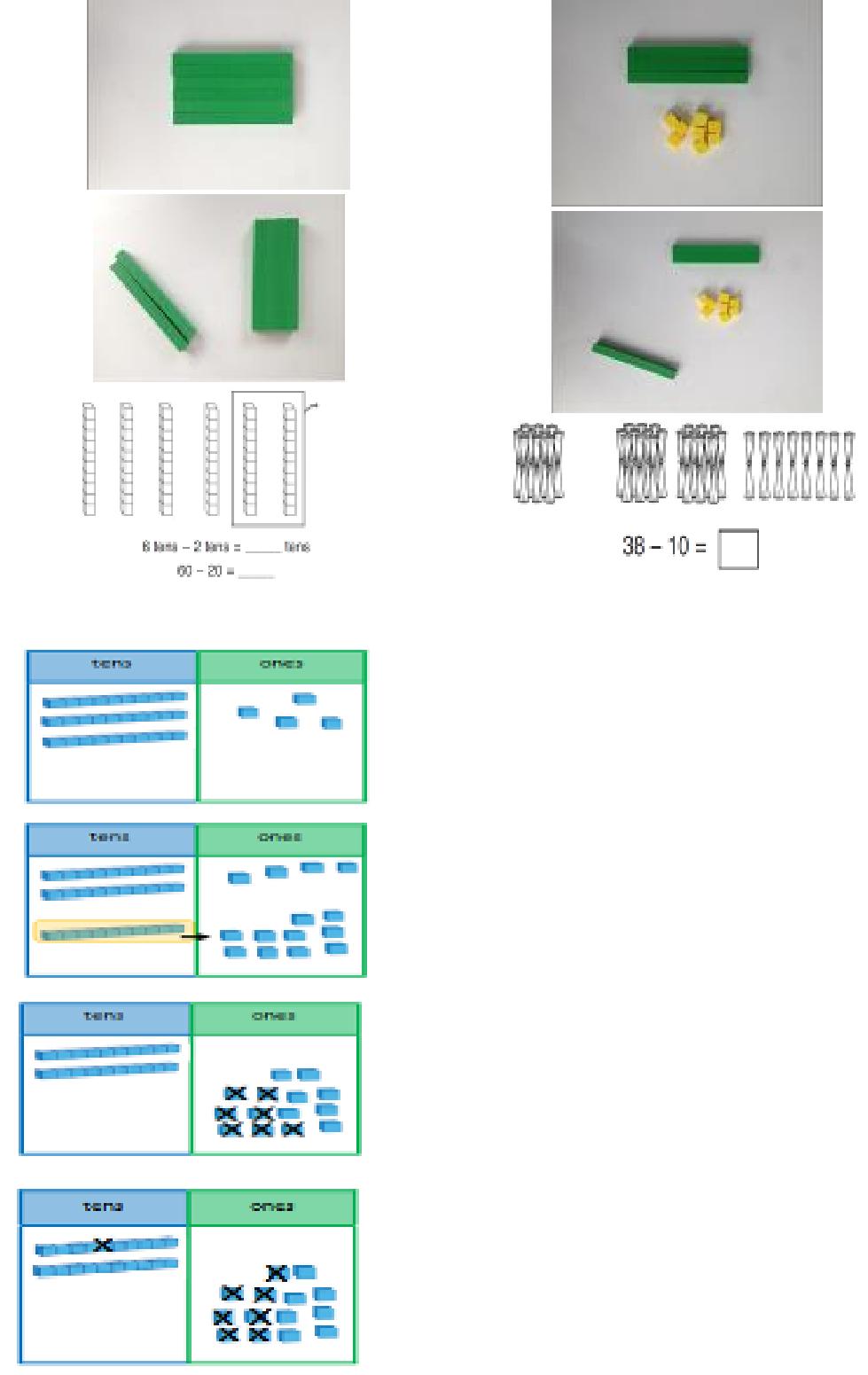


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|  |  |  |
| --- | --- | --- |
| **Subtracting** | 40 = 60 –20 | 38 −10 = 28 |
| **multiples of ten** |  |  |
| *Using the vocabulary* |  |  |
| *of 1 ten, 2 tens, 3 tens* |  |  |
| *etc. alongside 10, 20,* |  |  |
| *30 is important as* |  |  |
| *pupils need to* |  |  |
| *understand that it is a* |  |  |
| ***ten*** *not a one that is* |  |  |
| *being taken away.* |  |  |
|  |  |  |
| **Column method** |  | 34 −17 = 17 |
| **with regrouping** |  |  |
| *This example shows* |  |  |
| *how pupils should* |  |  |
| *work practically when* |  |  |
| *being introduced to* |  |  |
| *this method.* |  |  |
| *There is no formal* |  |  |
| *recording in columns* |  |  |
| *in Year 1 but this* |  |  |
| *practical work will* |  |  |
| *prepare pupils for* |  |  |
| *formal methods in* |  |  |
| *Year 2.* |  |  |
| *See additional* |  |  |
| *guidance on unit* |  |  |
| *pages to support with* |  |  |
| *this method.* |  |  |
|  |  |  |



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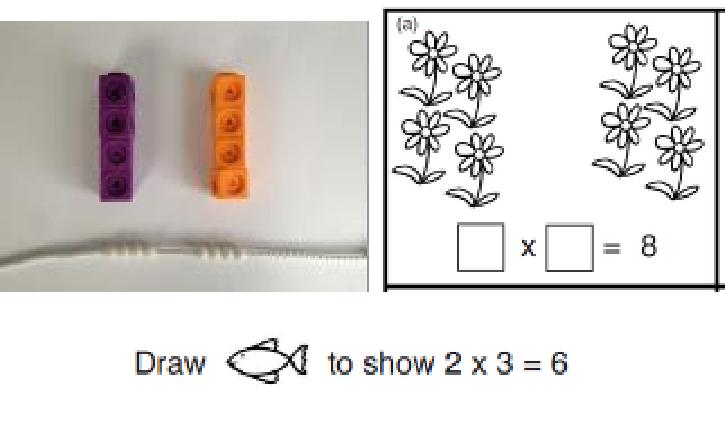
**National Curriculum objectives linked to multiplication and division**

**These objectives are explicitly covered through the strategies outlined in this document:**

* Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

**Teachers should refer to definitions and guidance on the**  [**structures for multiplicatio**](#page8)**n**  [**and division**](#page8) **to provide a range of appropriate real-life contexts for calculations.**

|  |  |  |
| --- | --- | --- |
|  | **Y1 Multiplication** |  |
|  |  |  |
| **Strategy & guidance** | **CPA** |  |
|  |  |  |
| **Skip counting in** |  |  |
| **multiples of 2, 5, 10 from** |  |  |
| **zero** |  |  |
| *The representation for the* |  |  |
| *amount of groups supports* |  |  |
| *pupils’ understanding of the* |  |  |
| *written equation. So two* | 4 × 5 = 20 |  |
| *groups of 2 are 2, 4. Or five* |  |
|  |  |
| *groups of 2 are 2, 4, 6, 8, 10.* |  |  |
| *Count the groups as pupils* |  |  |
| *are skip counting.* |  |  |
| *Number lines can be used in* |  |  |
| *the same way as the bead* |  |  |
| *string.* |  |  |
| *Pupils can use their fingers* |  |  |
| *as they are skip counting.* | 2 × 4 = 8 |  |
|  |  |
|  |  |  |
| **Making equal groups** |  |  |
| **and counting the total** |  |  |
| *How this would be* |  |  |
| *represented as an equation* |  |  |
| *will vary. This could be 2 × 4* |  |  |
| *or 4 × 2. The importance* |  |  |
| *should be placed on the* |  |  |
| *vocabulary used alongside* |  |  |
| *the equation. So this picture* |  |  |
| *could represent 2 groups of* |  |  |
| *4 or 4 twice.* |  |  |
|  |  |  |

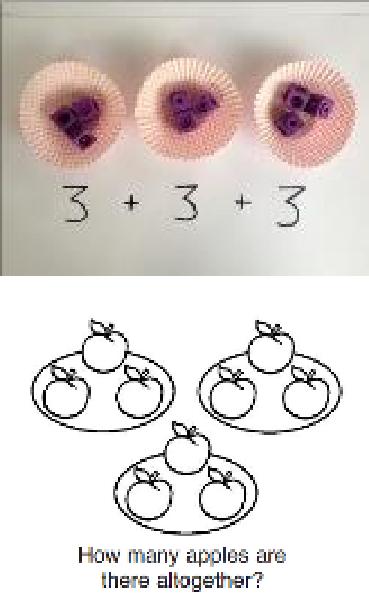


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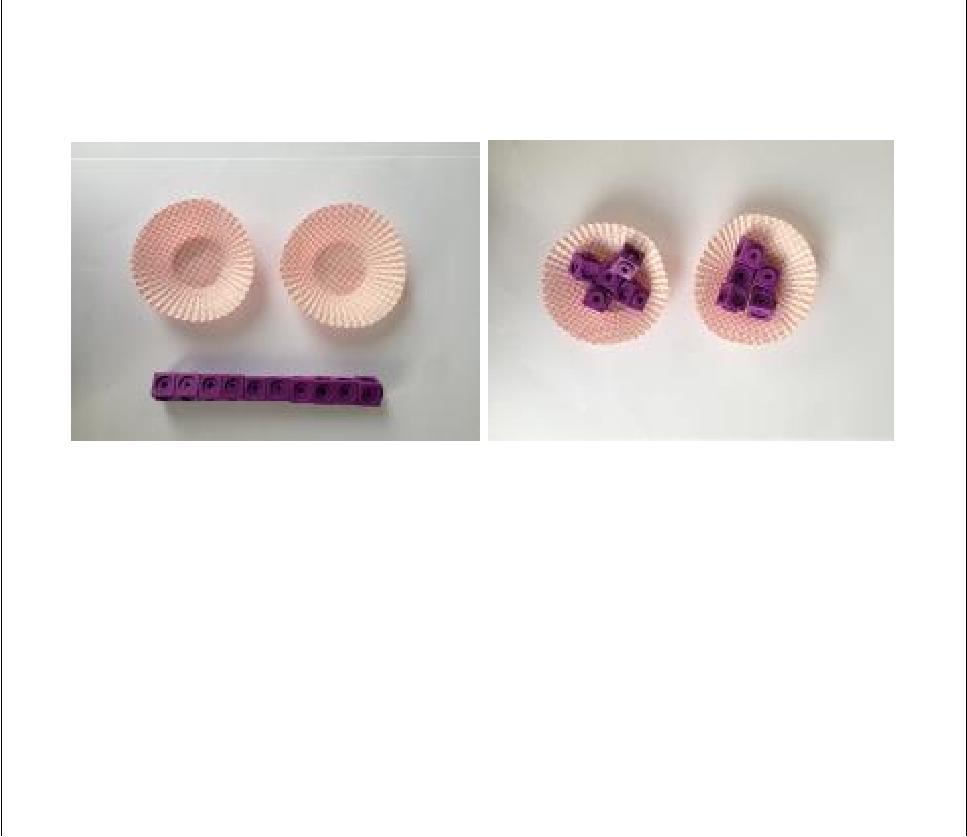
21



|  |  |
| --- | --- |
| **Solve multiplications** | 3 x 3 = 3 + 3 + 3 |
| **using repeated addition** |  |
| *This strategy helps pupils* |  |
| *make a clear link between* |  |
| *multiplication and division* |  |
| *as well as exemplifying the* |  |
| *‘repeated addition’ structure* |  |
| *for multiplication. It is a* |  |
| *natural progression from* |  |
| *the previous ‘count all’* |  |
| *strategy as pupils can be* |  |
| *encouraged to ‘count on’.* |  |
| *However, as number bonds* |  |
| *knowledge grows, pupils* |  |
| *should rely more on these* |  |
| *important facts to calculate* |  |
| *efficiently.* |  |
|  | 3 + 3 + 3 = 9 |
|  |  |



|  |  |
| --- | --- |
|  | **Y1 Division** |
|  |  |
| **Strategy & guidance** | **CPA** |
|  |  |
| **Sharing objects into** | 10 ÷ 2 = 5 |
| **groups** |  |



*Pupils should become familiar with division equations through working practically.*

*The division symbol is not formally taught at this stage.*



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Progression in calculations Year 2

**National Curriculum objectives linked to addition and subtraction**

**These objectives are explicitly covered through the strategies outlined in this document:**

* Add and subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones; a two-digit number and tens; 2 two-digit numbers; adding three one-digit numbers.
* Add and subtract numbers mentally, including: a three-digit number and ones; a three-digit number and tens; a three-digit number and hundreds (Year 3).
* Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100.
* Find 10 or 100 more or less than a given number (Year 3).
* Show that addition of two numbers can be done in any order (commutative) but subtraction of one number from another cannot.
* Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.
* Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction (Year 3).

**The following objectives should be planned for lessons where new strategies are being introduced and developed:**

* Solve problems with addition and subtraction: using concrete objects and pictorial representations, including those involving numbers, quantities and measures; apply increasing knowledge of mental and written methods.
* Solve problems, including missing number problems, using number facts, place value and more complex addition and subtraction. (Year 3)

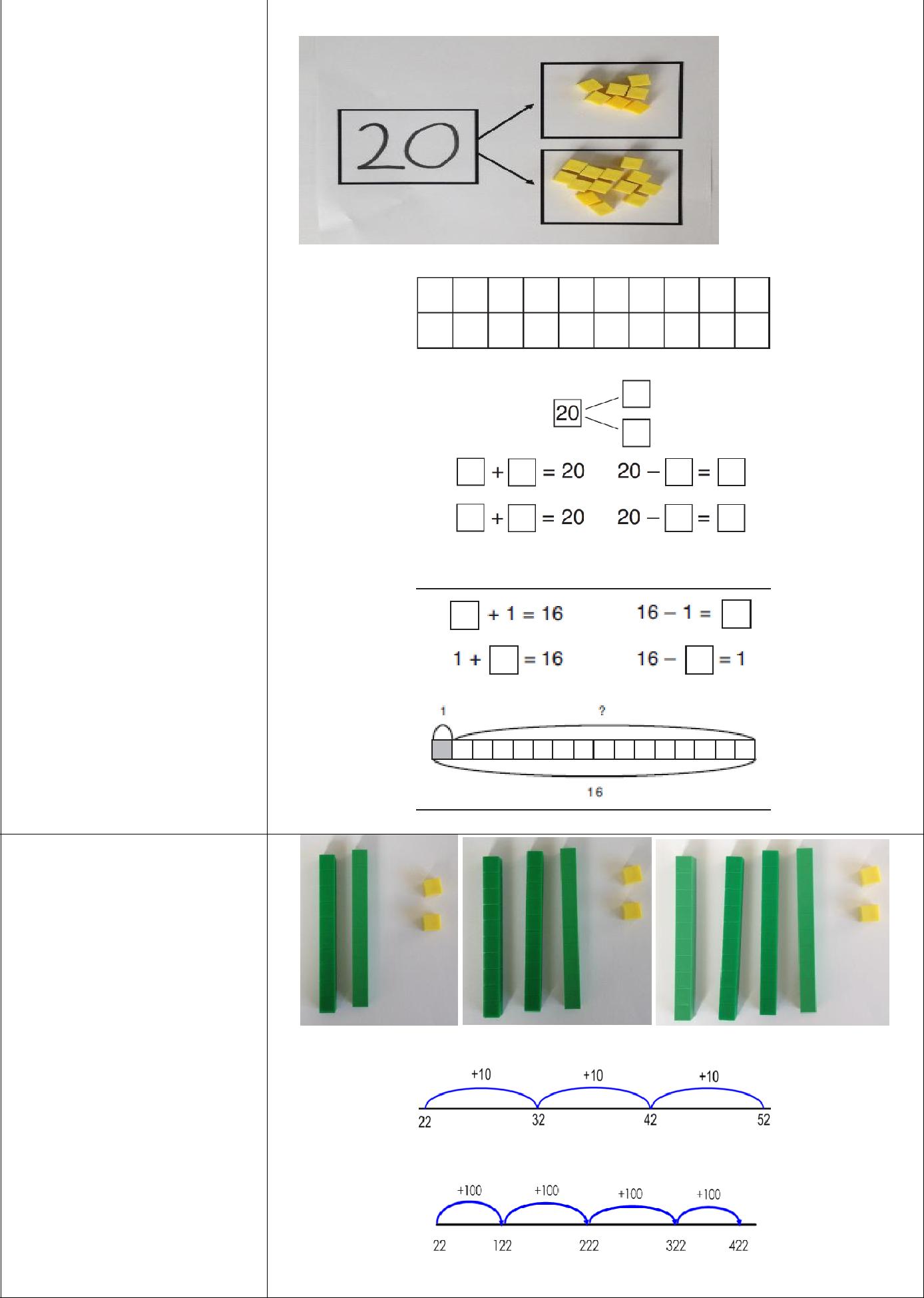
**Teachers should refer to the definitions and guidance on the**  [**structures for addition an**](#page5)**d**  [**subtraction**](#page5) **to provide a range of appropriate real-life contexts for calculations.**

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|  |  |  |
| --- | --- | --- |
|  | **Y2 Addition** |  |
|  |  |  |
| **Strategy & guidance** | **CPA** |  |
|  |  |  |
| **Part-part-whole** |  |  |
| *Pupils explore the different* | 20 = 17 + 3 |  |
| *ways of making 20. They* |  |
| 20 = 3 + 17 |  |
| *can do this with all* |  |
| 20 –3 = 17 |  |
| *numbers using the same* |  |
|  |  |
| *representations.* | 20 –17 = 3 |  |
| *This model develops* |  |  |
| *knowledge of the inverse* |  |  |
| *relationship between* |  |  |
| *addition and subtraction* |  |  |
| *and is used to find the* |  |  |
| *answer to missing number* |  |  |
| *problems.* |  |  |



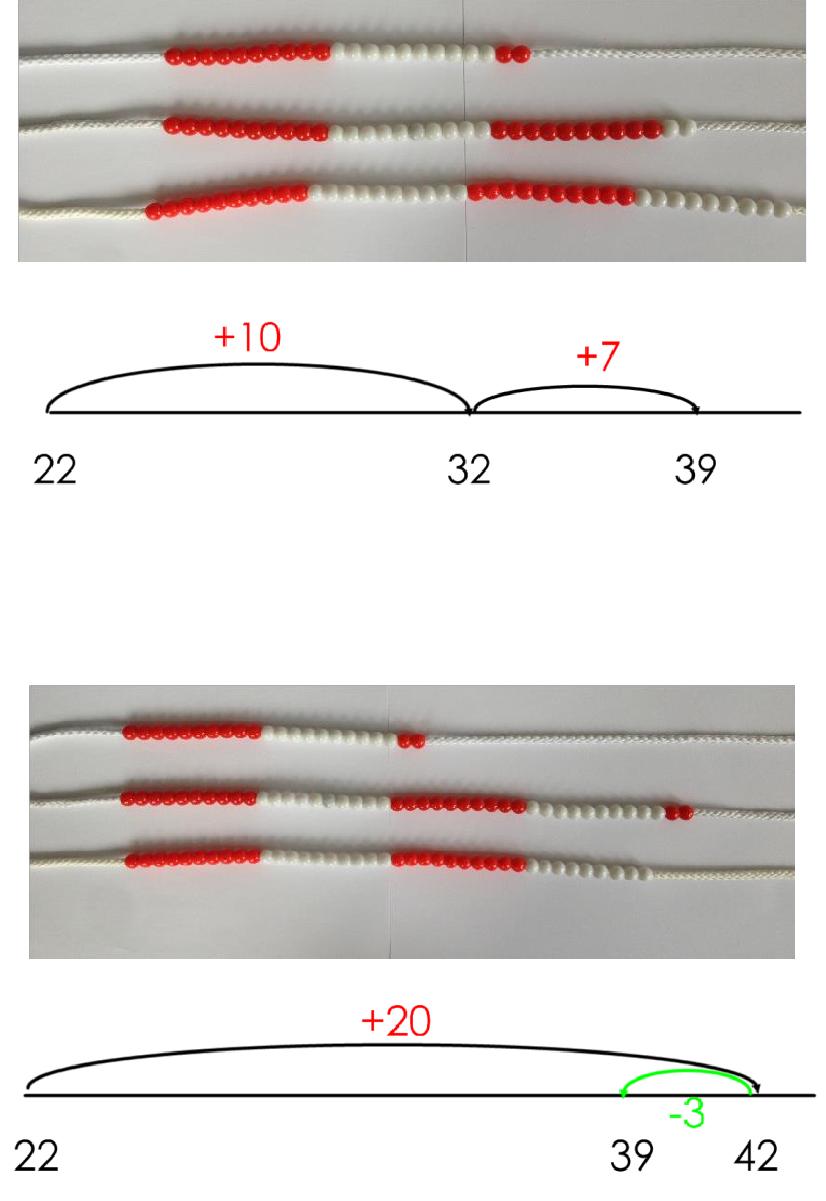
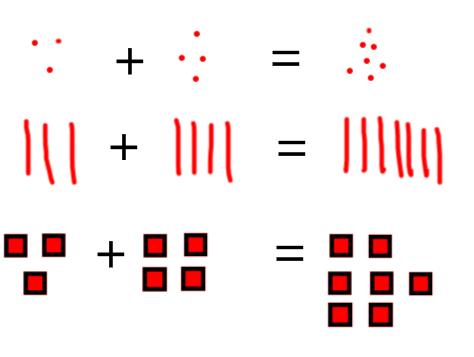
**Counting on in tens and hundreds**

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|  |  |  |
| --- | --- | --- |
| **Strategy & guidance** | **CPA** |  |
|  |  |  |
| **Using known facts to** | 3 + 4 = 7 |  |
| **create derived facts** |  |  |
|  | *leads to* |  |
| *Dienes blocks should be* |  |  |
| *used alongside pictorial* | 30 + 40 = 70 |  |
| *and abstract* |  |  |
| *representations when* | *leads to* |  |
| *introducing this strategy.* | 300 + 400 = 700 |  |
|  |  |
|  |  |  |
| **Partitioning one** |  |  |
| **number, then adding** |  |  |
| **tens and ones** |  |  |
| *Pupils can choose* |  |  |
| *themselves which of the* |  |  |
| *numbers they wish to* |  |  |
| *partition. Pupils will begin* |  |  |
| *to see when this method is* |  |  |
| *more efficient than adding* |  |  |
| *tens and taking away the* |  |  |
| *extra ones, as shown.* |  |  |
|  | 22 + 17 = 39 |  |
|  |  |  |
| **Round and adjust** |  |  |
| **(sometimes known as a** |  |  |
| **compensating strategy)** |  |  |
| *Pupils will develop a sense* |  |  |
| *of efficiency with this* |  |  |
| *method, beginning to see* |  |  |
| *when rounding and* |  |  |
| *adjusting is more efficient* |  |  |
| *than adding tens and then* |  |  |
| *ones.* |  |  |
|  | 22 + 17 = 39 |  |
|  |  |  |



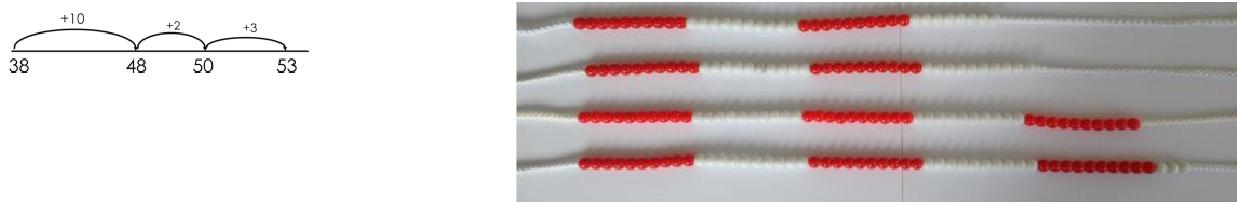
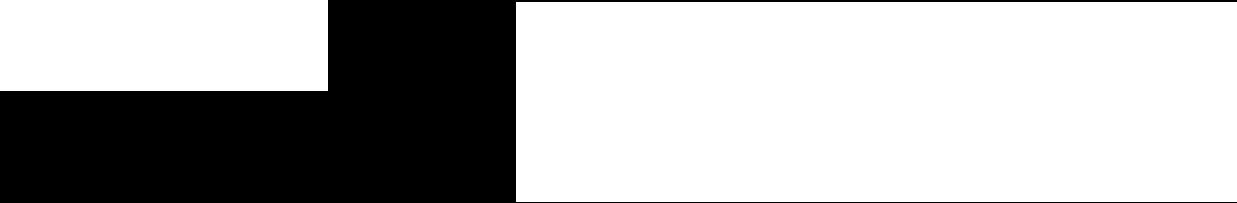
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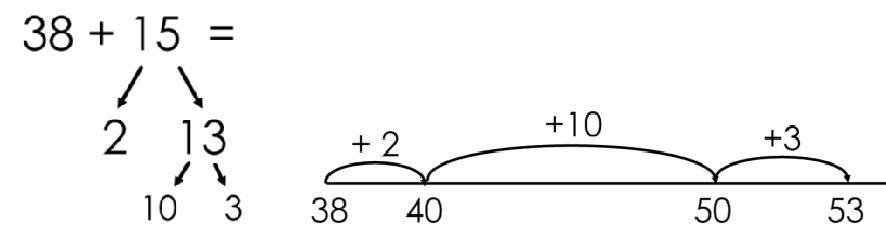


|  |  |
| --- | --- |
| **Strategy & guidance** | **CPA** |

**Make ten strategy**

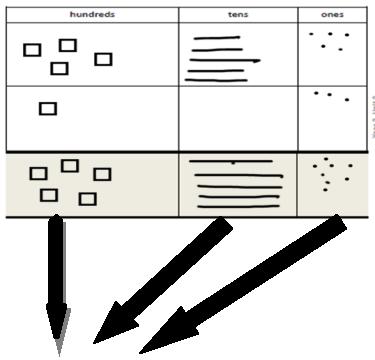


*How pupils choose to apply this strategy is up to them; however, the focus should always be on efficiency.*

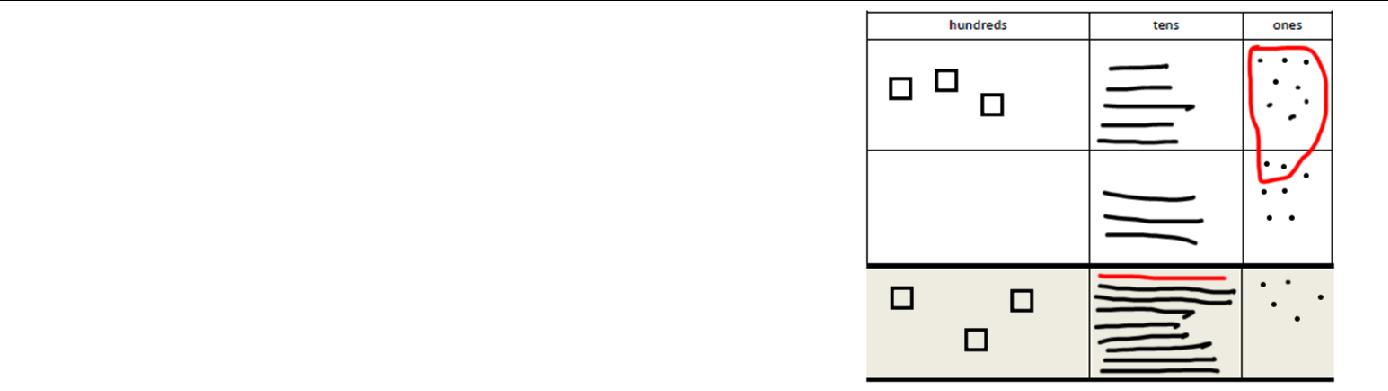


*It relies on an understanding that numbers can be partitioned in different ways in order to easily make a multiple of ten.*

**Partitioning to add without regrouping**

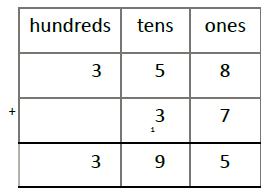


|  |  |  |
| --- | --- | --- |
| *As in Year 1, this is a* |  |  |
| *mental strategy rather* |  |  |
| *than a formal written* |  |  |
| *method. Pupils use the* |  |  |
| *Dienes blocks (and later,* |  |  |
| *images) to represent 3-* |  |  |
| *digit numbers but do not* |  |  |
| *record a formal written* |  |  |
| *method if there is no* | 4 5 5 + 1 0 3 = 5 5 8 |  |
| *regrouping.* |  |



**Column method with regrouping**

*Dienes blocks should be used alongside the pictorial representations; they can be placed on the place value grid before pupils make pictorial representations.*



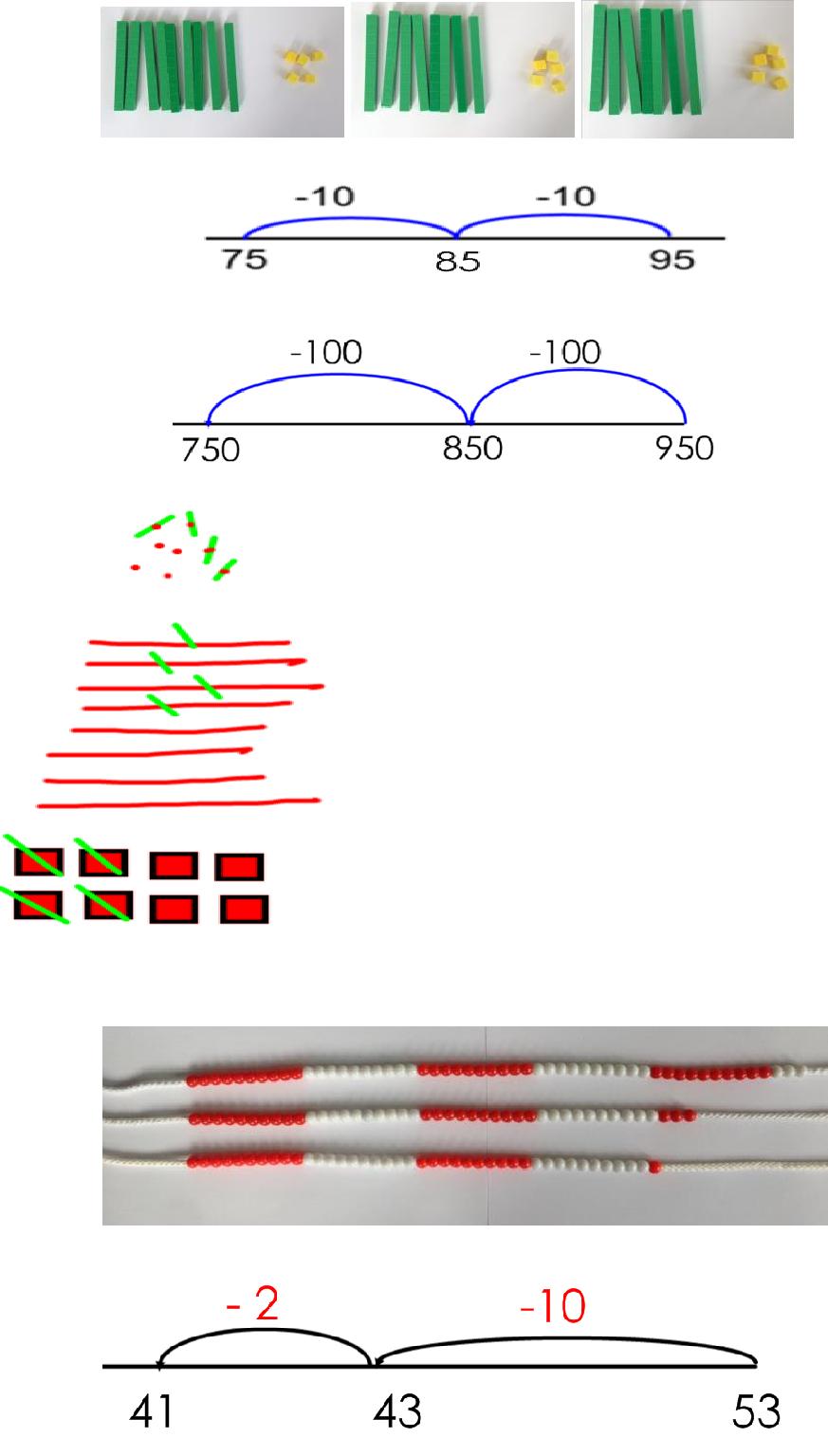
*As in Year 1, the focus for the column method is to develop a strong understanding of place value.*

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|  |  |  |
| --- | --- | --- |
|  | **Y2 Subtraction** |  |
|  |  |  |
| **Strategy & guidance** | **CPA** |  |
|  |  |  |
| **Counting back in** |  |  |
| **multiples of ten and one** |  |  |
| **hundred** |  |  |
|  |  |  |
| **Using known number** |  |  |
| **facts to create derived** |  |  |
| **facts** | 8 −4 = 4 |  |
| *Dienes blocks should be* | *leads to* |  |
| *used alongside pictorial* |  |  |
| *and abstract* | 80 −40 = 40 |  |
| *representations when* | *leads to* |  |
| *introducing this strategy,* |  |
|  |  |
| *encouraging pupils to* | 800 −400 = 400 |  |
| *apply their knowledge of* |  |
|  |  |
| *number bonds to add* |  |  |
| *multiples of ten and 100.* |  |  |
|  |  |  |
| **Subtracting tens and** | 53 −12 = 41 |  |
| **ones** |  |  |
| *Pupils must be taught to* |  |  |
| *partition the second* |  |  |
| *number for this strategy as* |  |  |
| *partitioning both numbers* |  |  |
| *can lead to errors if* |  |  |
| *regrouping is required.* |  |  |
|  |  |  |

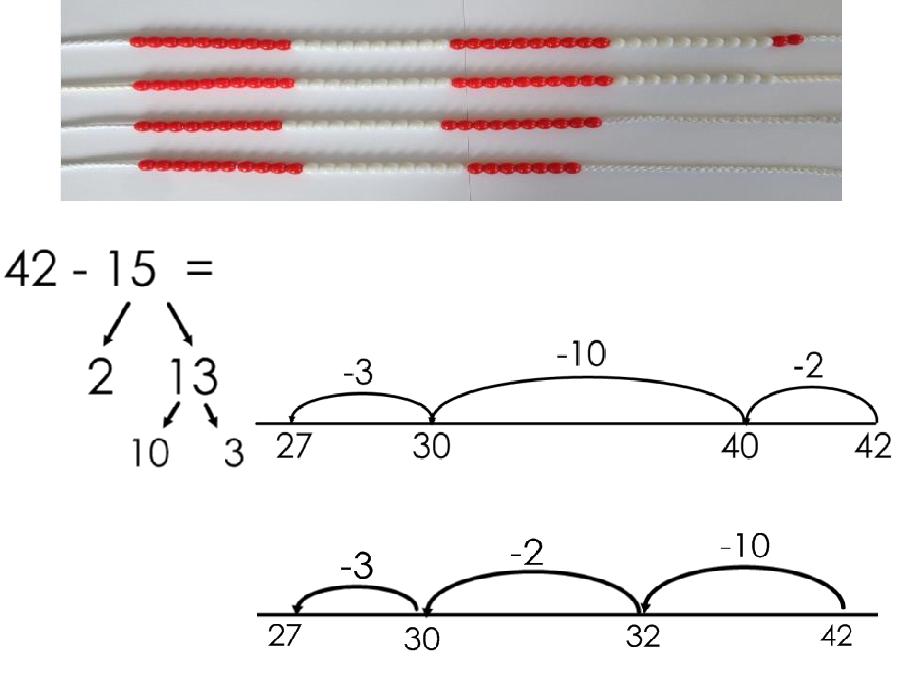
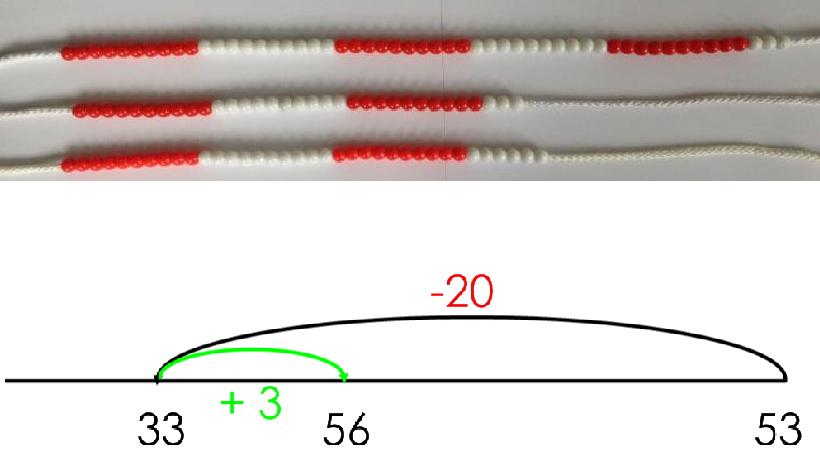


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|  |  |
| --- | --- |
| **Strategy & guidance** | **CPA** |
|  |  |
| **Round and adjust** |  |
| **(sometimes known as a** |  |
| **compensating strategy)** |  |
| *Pupils must be taught to* |  |
| *round the number that is* |  |
| *being subtracted.* |  |
| *Pupils will develop a sense* |  |
| *of efficiency with this* |  |
| *method, beginning to* |  |
| *identify when this method* |  |
| *is more efficient than* |  |
| *subtracting tens and then* |  |
| *ones.* |  |
|  | 53 −17 = 36 |
|  |  |
| **Make ten** |  |
| *How pupils choose to apply* |  |
| *this strategy is up to them.* |  |
| *The focus should always be* |  |
| *on efficiency.* |  |
| *It relies on an* |  |
| *understanding that* |  |
| *numbers can be partitioned* |  |
| *in different ways in order* |  |
| *to subtract to a multiple of* |  |
| *ten.* |  |
| *Pupils should develop an* |  |
| *understanding that the* |  |
| *parts can be added in any* |  |
| *order.* |  |
|  |  |

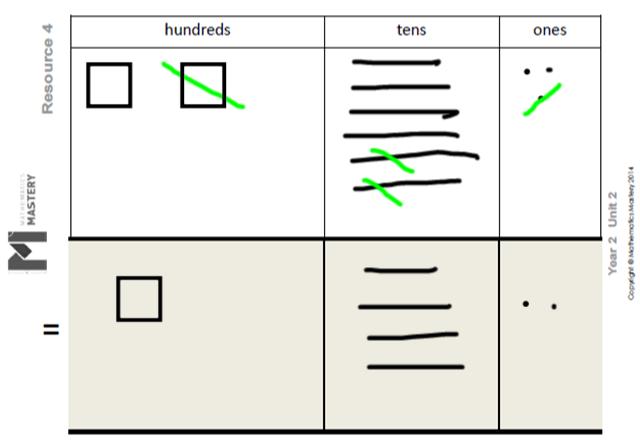
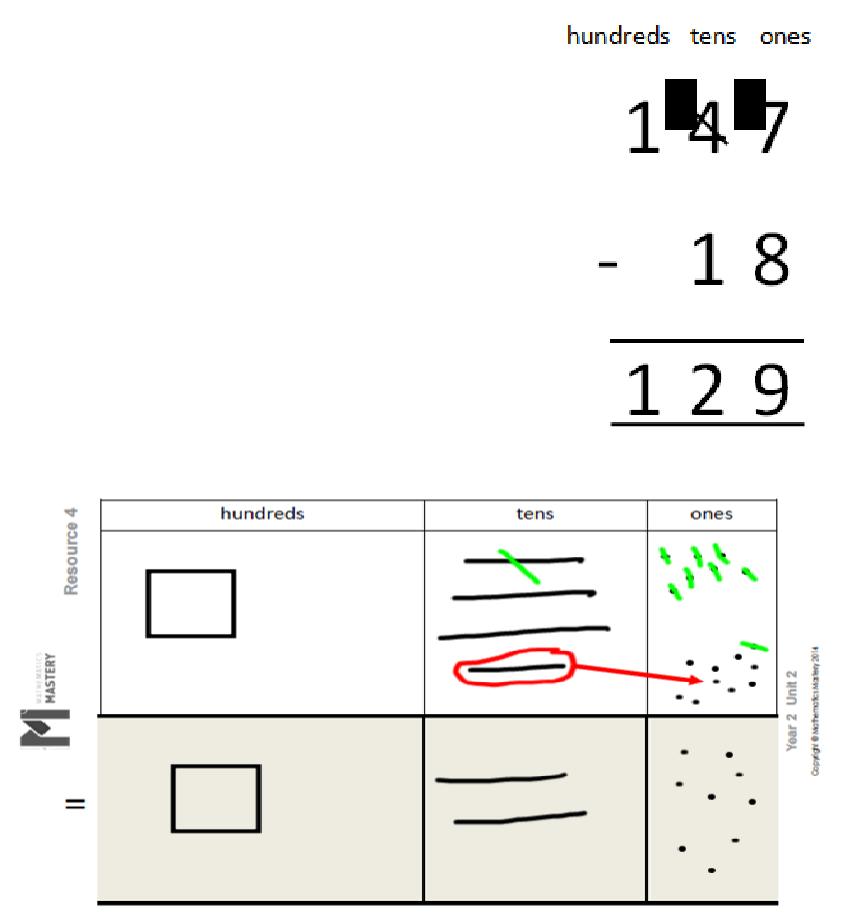


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|  |  |  |
| --- | --- | --- |
| **Strategy & guidance** | **CPA** |  |
|  |  |  |
| **Partitioning to subtract** |  |  |
| **without regrouping** |  |  |
| *As in Year 1, the focus is to* |  |  |
| *develop a strong* |  |  |
| *understanding of place* |  |  |
| *value and pupils should* |  |  |
| *always be using concrete* |  |  |
| *manipulatives alongside* |  |  |
| *the pictorial.* |  |  |
| *Formal recording in* |  |  |
| *columns is unnecessary for* |  |  |
| *this mental strategy. It* |  |  |
| *prepares them to subtract* | 263 −121= 142 |  |
| *with 3-digits when* |  |
|  |  |
| *regrouping is required.* |  |  |
|  |  |  |
| **Column method with** |  |  |
| **regrouping** |  |  |
| *The focus for the column* |  |  |
| *method is to develop a* |  |  |
| *strong understanding of* |  |  |
| *place value and concrete* |  |  |
| *manipulatives should be* |  |  |
| *used alongside.* |  |  |
| *Pupils are introduced to* |  |  |
| *calculations that require* |  |  |
| *two instances of* |  |  |
| *regrouping (initially from* |  |  |
| *tens to one and then from* |  |  |
| *hundreds to tens). E.g. 232* |  |  |
| *–157 and are given plenty* |  |  |
| *of practice using concrete* |  |  |
| *manipulatives and images* |  |  |
| *alongside their formal* |  |  |
| *written methods, ensuring* |  |  |
| *that important steps are* |  |  |
| *not missed in the recording.* |  |  |
| *Caution should be exercised* |  |  |
| *when introducing* |  |  |
| *calculations requiring* |  |  |
| *‘regrouping to regroup’* |  |  |
| *(e.g. 204 –137) ensuring* |  |  |
| *ample teacher modelling* |  |  |
| *using concrete* |  |  |
| *manipulatives and images.* |  |  |
|  |  |  |



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**National Curriculum objectives linked to multiplication and division**

**These objectives are explicitly covered through the strategies outlined in this document:**

* Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.
* Recall and use multiplication and division facts for the 3 and 4 multiplication tables (Year 3).
* Show that multiplication of two numbers can be done in any order (commutative) but division of one number by another cannot.

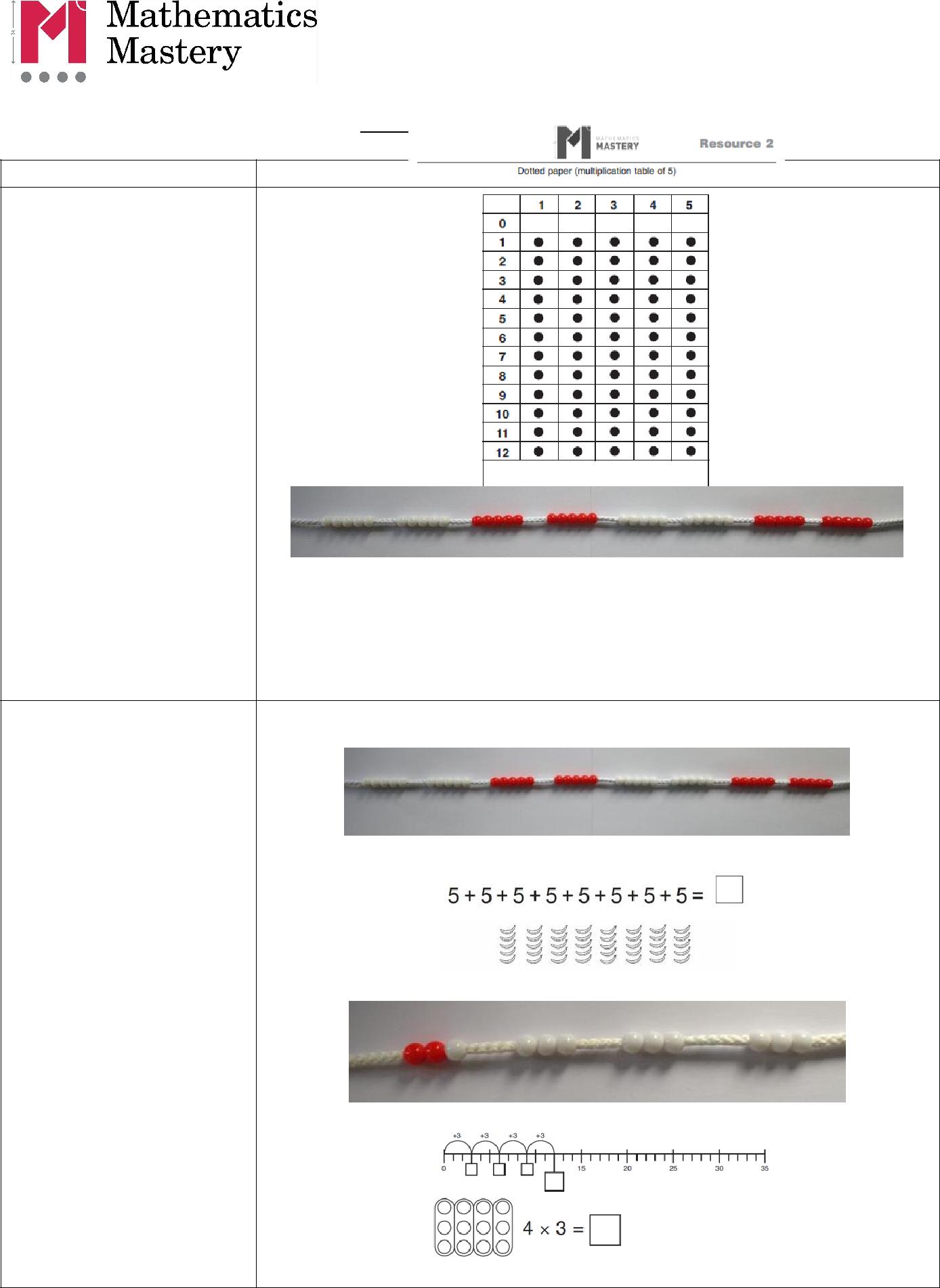
**The following objectives should be planned for lessons where new strategies are being introduced and developed:**

* Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equal (=) signs.
* Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods and multiplication and division facts, including problems in context.

**Teachers should refer to definitions and guidance on the**  [**structures for multiplicatio**](#page8)**n**  [**and division**](#page8) **to provide a range of appropriate real-life contexts for calculations.**

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**Y2**

**Strategy & guidance**

**Skip counting in multiples of 2, 3, 4, 5, 10 from zero**

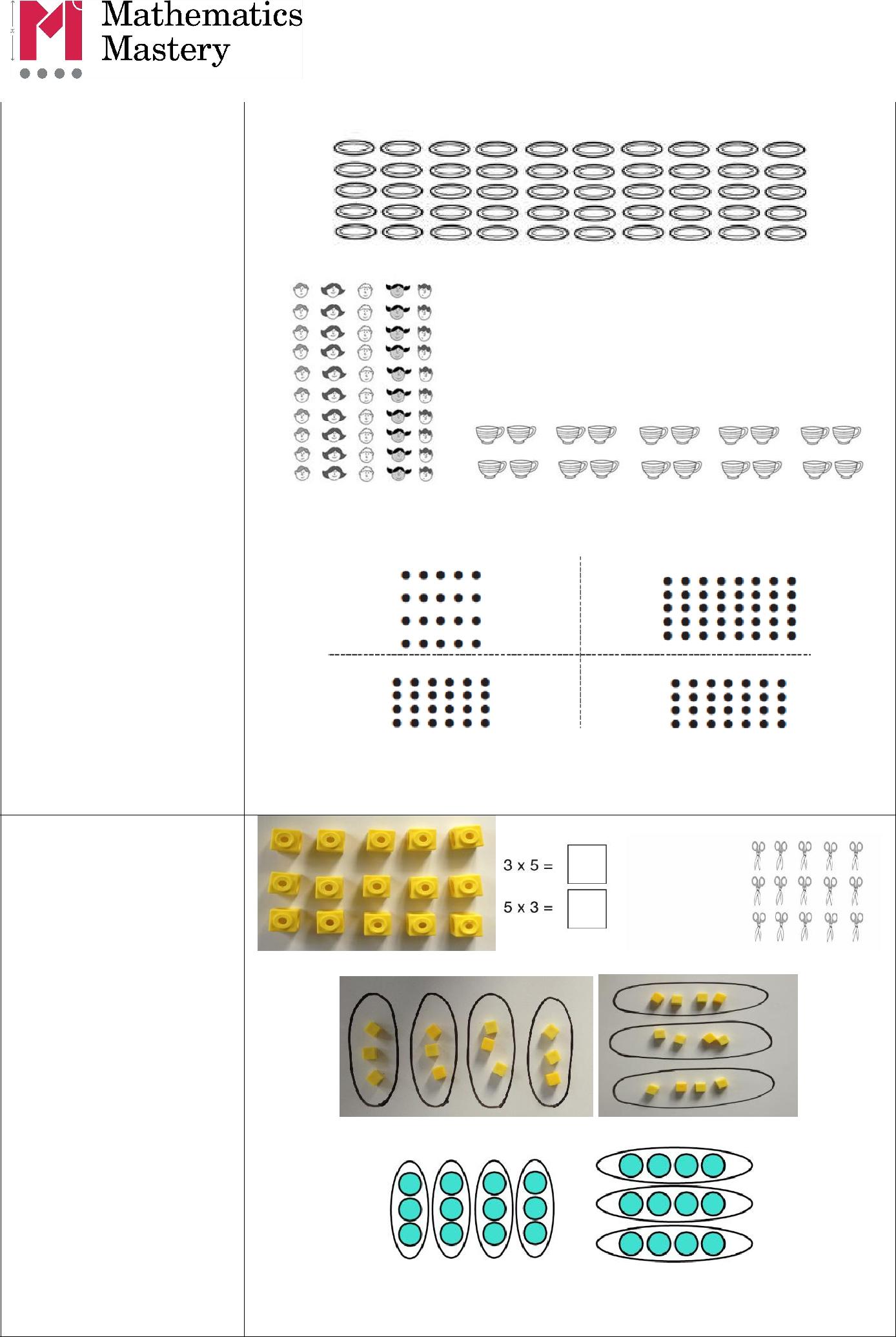
*Pupils can use their fingers as they are skip counting, to develop an understanding of ‘groups of’.*

**Multiplication as repeated addition**

*Pupils apply skip counting to help find the totals of repeated additions.*

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|  |  |
| --- | --- |
| **Strategy & guidance** | **CPA** |
|  |  |
| **Arrays to represent** |  |
| **multiplication** |  |
| **equations** |  |

*Concrete manipulatives and images of familiar objects begin to be organised into arrays and, later, are shown alongside dot arrays. It is important to discuss with pupils how arrays can be useful.*

*Pupils begin to understand multiplication in a more abstract fashion, applying their skip counting skills to identify the multiples of the 2x, 5x and 10x tables.*

*The relationship between multiplication and division also begins to be demonstrated.*

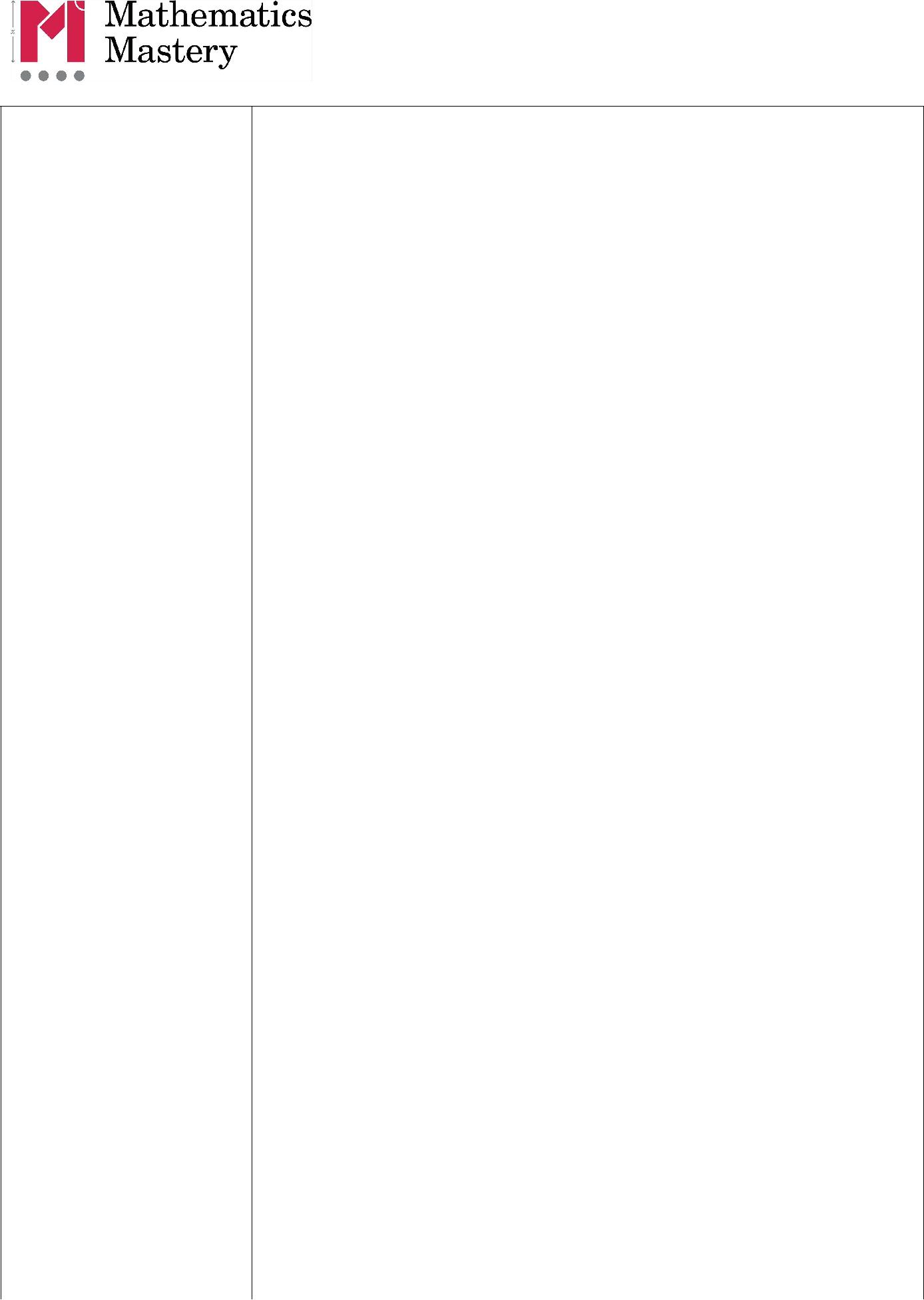
**Multiplication is commutative**

*Pupils should understand that an array and, later, bar models can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.*

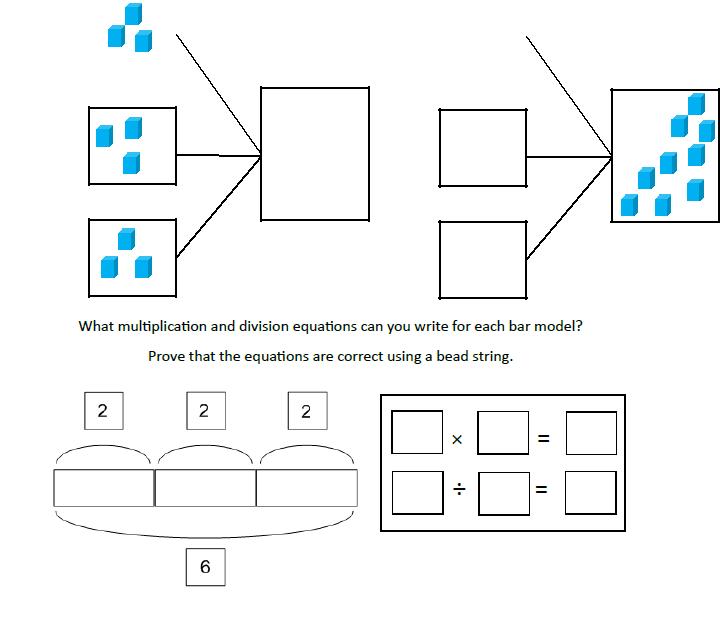
12 = 3 × 4 12 = 4 × 3

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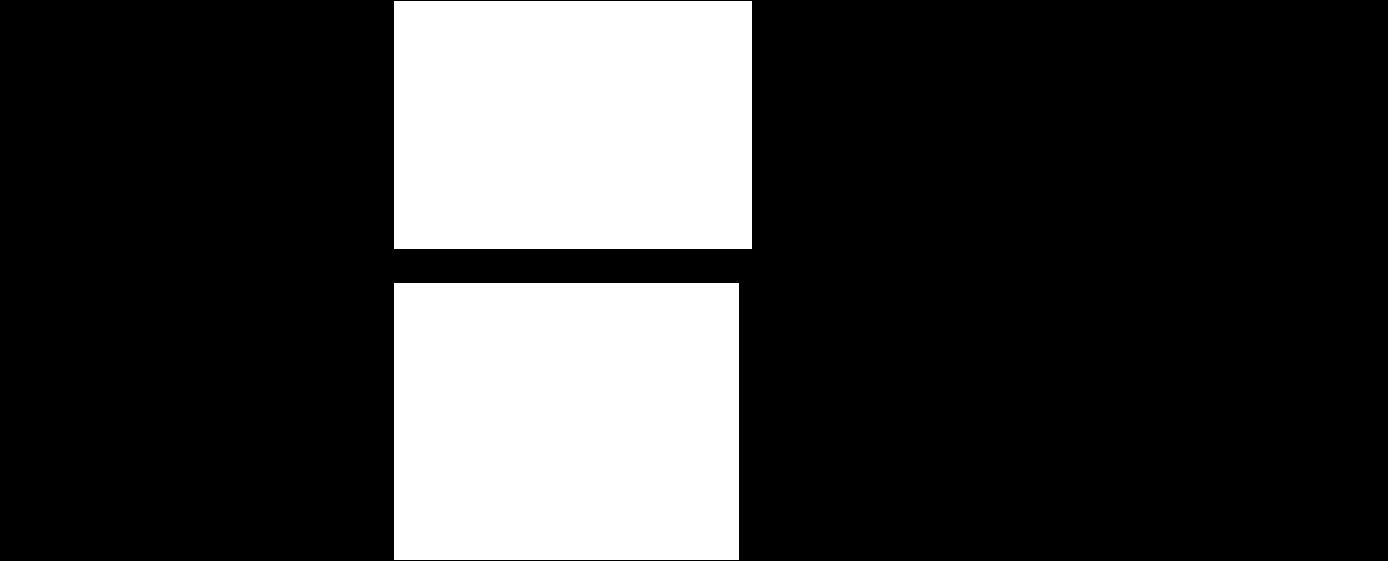


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| **Strategy & guidance** |  |  |  | **CPA** | | | | |  |
|  |  |  |  |  |  |  |  |  |  |
| **Use of part-part-** | There are three equal parts. Each part has a value of three. What is the whole? | | | | | | | |  |
| **whole model to** | **3 × 3 =** | |  | **9 ÷ 3 =** | | |  |  |  |
| **establish the inverse** |  |  |  |  |
| **relationship between** |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
| **multiplication and** |  |  |  |  |  |  |  |  |  |
| **division** |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |



*This link should be made explicit from early on, using the language of the part-part-whole model, so that pupils develop an early understanding of the relationship between multiplication and division. Bar models (with Cuisenaire rods) should be used to identify the whole, the value of the parts and the number of parts.*

*It is important to highlight that with multiplication, the parts are of equal value as this is different to how this model is used for addition and subtraction.*



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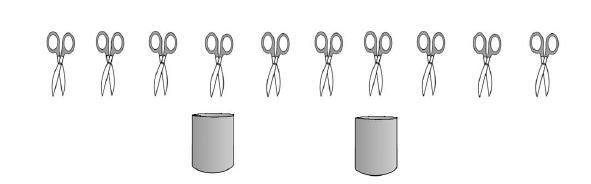


|  |  |
| --- | --- |
|  | **Y2 Division** |
|  |  |
| **Strategy & guidance** | **CPA** |
|  |  |
| **Division as sharing** | 10 ÷ 2 = 5 |

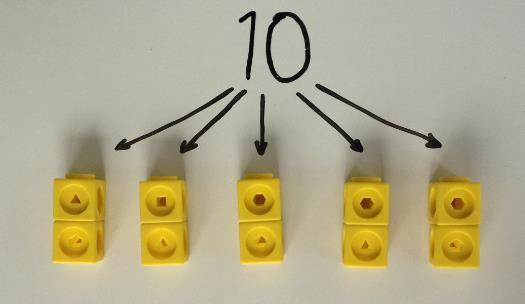


*Here, division is shown as sharing.*

*If we have ten pairs of scissors and we share them between two pots, there will be 5 pairs of scissors in each pot.*

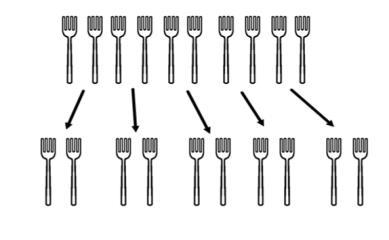


|  |  |
| --- | --- |
| **Division as grouping** | 10 ÷ 2 = 5 |



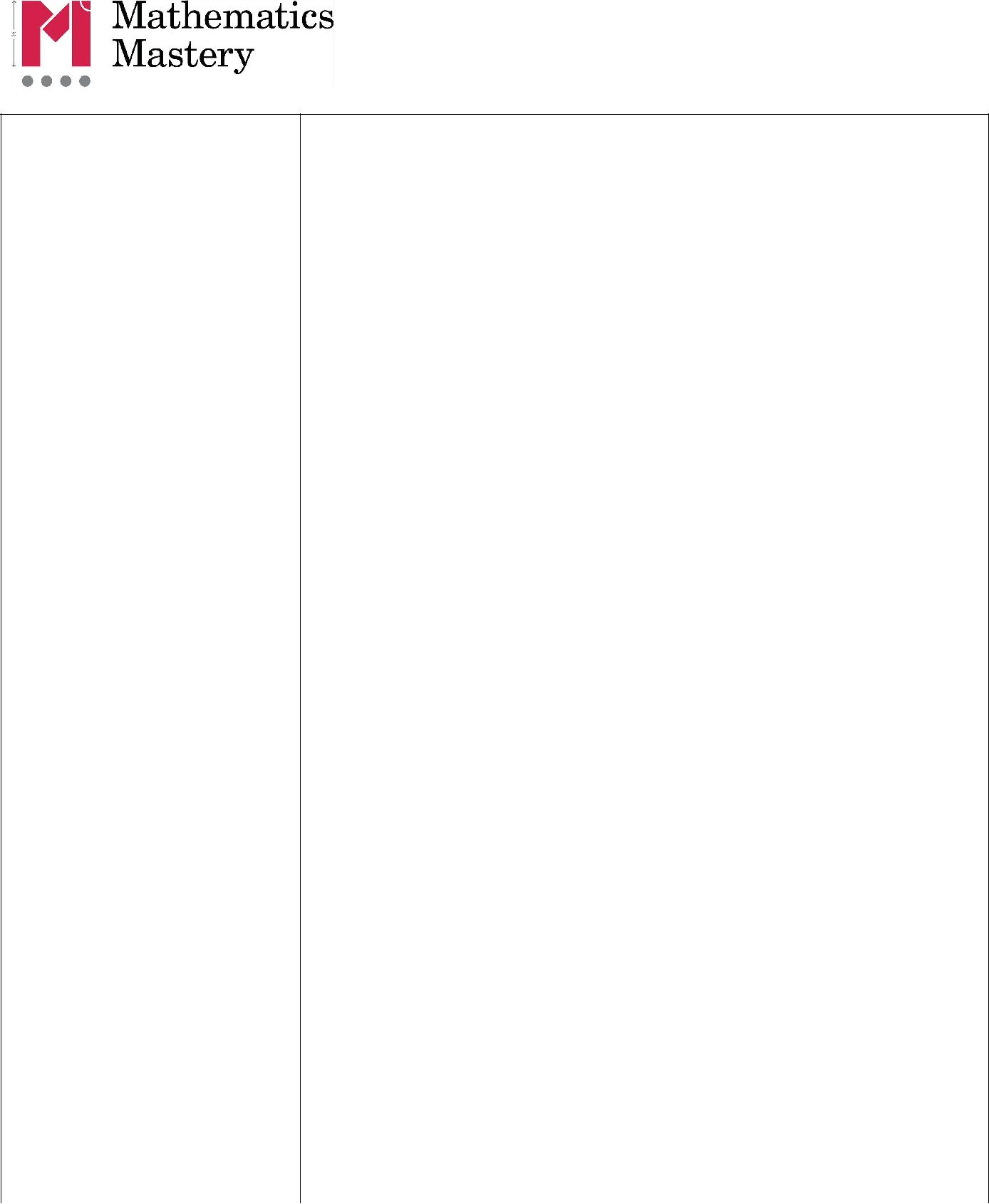
*Here, division is shown as grouping.*

*If we have ten forks and we put them into groups of two, there are 5 groups.*

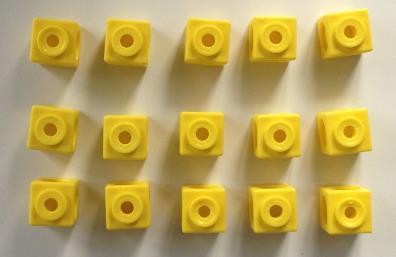


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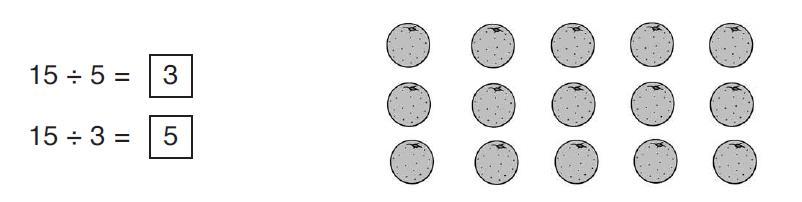
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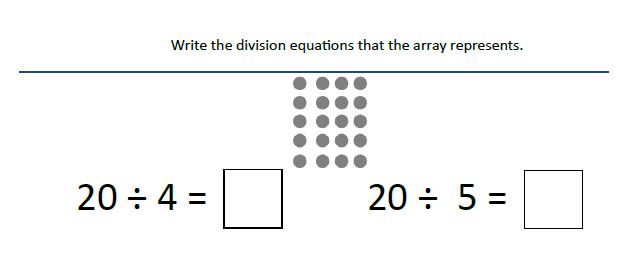
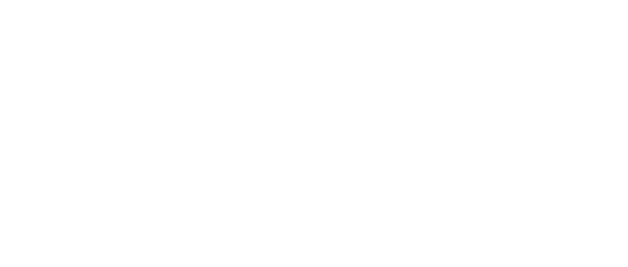
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| **Strategy & guidance** | **CPA** |
|  |  |
| **Use of part-part-whole** |  |
| **model to represent** |  |
| **division equations and** |  |
| **to emphasise the** |  |
| **relationship between** |  |
| **division and** |  |
| **multiplication** |  |



*Pupils use arrays of concrete manipulatives and images of familiar objects to solve division equations.*



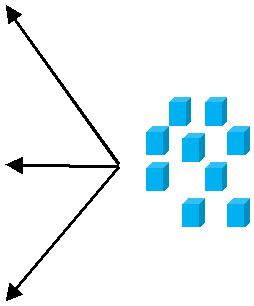
*They begin to use dot arrays to develop a more abstract concept of division.*



*It is important to highlight that with multiplication and division, the parts are of equal value as this is different to how this model is used for addition and subtraction.*

The whole is nine. There are three equal parts. What is the value of

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| each part? |  | **9 ÷ 3 =** | |  |  |  |
|  |  |
|  |  |  |  |  |  |  |
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|  |  |  |  |  |  |  |
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Progression in calculations Year 3

**National Curriculum objectives linked to addition and subtraction**

**These objectives are explicitly covered through the strategies outlined in this document:**

* add and subtract numbers mentally, including:

o a three-digit number and ones

o a three-digit number and tens

o a three-digit number and hundreds

* add and subtract numbers with up to four digits, using formal written methods of columnar addition and subtraction (four digits is Year 4)
* find 10 or 100 more or less than a given number
* find 1000 more or less than a given number (Year 4)
* estimate the answer to a calculation and use inverse operations to check answers

**The following objectives should be planned for lessons where new strategies are being introduced and developed:**

* solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction

**Teachers should refer to definitions and guidance on the**  [**structures for additio**](#page5)**n**  [**and subtraction**](#page5) **to provide a range of appropriate real-life contexts for calculations.**

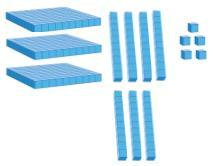
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**Y3 Addition & Subtraction**

|  |  |  |  |
| --- | --- | --- | --- |
| **Strategy & guidance** |  | **CPA** |  |
|  |  | |  |
| **Add and subtract numbers mentally,** | It is important to model the mental strategy | |  |
| **including:** | using concrete manipulatives in the first | |  |
|  |  |
|  **a three-digit number and ones;** | instance and pupils should be able to | |  |
|  |  |  |
|  | exemplify their own strategies using | |  |
|  **a three-digit number and tens;** | manipulatives if required, with numbers | |  |
|  **a three-digit number and hundreds** | appropriate to the unit they are working on | |  |
| (3-digit numbers in Units 1 & 4; 4-digit | |  |
|  |  |
| *Pupils learn that this is an appropriate strategy when* | numbers in Unit 13). However, pupils | |  |
| *they are able to use known and derived number facts* | should be encouraged to use known facts to | |  |
| *or other mental strategies to complete mental* |  |
| derive answers, rather than relying on | |  |
| *calculations with accuracy.* |  |
| counting manipulatives or images. | |  |
|  |  |
| *To begin with, some pupils will prefer to use this* |  |  |  |
| *strategy only when there is no need to regroup, using* | No regrouping |  |  |
| *number facts within 10 and derivations. More* | 345 + 30 | 274 - 50 |  |
| *confident pupils might choose from a range of mental* |  |
| *strategies that avoid written algorithms, including* | 1128 + 300 | 1312 - 300 |  |
| *(but not exhaustively):* |  |
|  |  |  |
|  *known number facts within 20,* | 326 + 342 | 856 - 724 |  |
|  *derived number facts,* |  | I know 4 + 3 = 7, |  |
|  *‘Make ten’,* |  | so 4 tens plus 3 |  |
|  | tens is equal to 7 |  |
|  |  |  |
|  *round and adjust* |  | tens. |  |
| *See Year 2 guidance for exemplification of these –the* |  | 345 + 30 = 375. |  |
|  |  |  |
| *use of concrete manipulatives other than Dienes* | With some regrouping | |  |
| *blocks is important in reinforcing the use of these* |  |  |  |
| *strategies.* | 416 + 25 | 232 - 5 |  |
| *It is important that pupils are given plenty of* | 383 + 130 | 455 - 216 |  |
| *(scaffolded) practice at choosing their own strategies* |  |
|  |  |  |
| *to complete calculations efficiently and accurately.* | 611 + 194 | 130 - 40 |  |
| *Explicit links need to be made between familiar* |  |
|  |  |  |
| *number facts and the calculations that they can be* | 1482 + 900 | 2382 - 500 |  |
| *useful for and pupils need to be encouraged to aim for* |  |  |  |
| *efficiency.* |  |  |  |
|  |  |  |  |

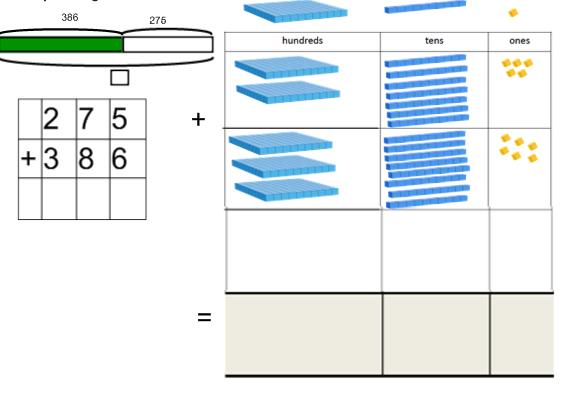


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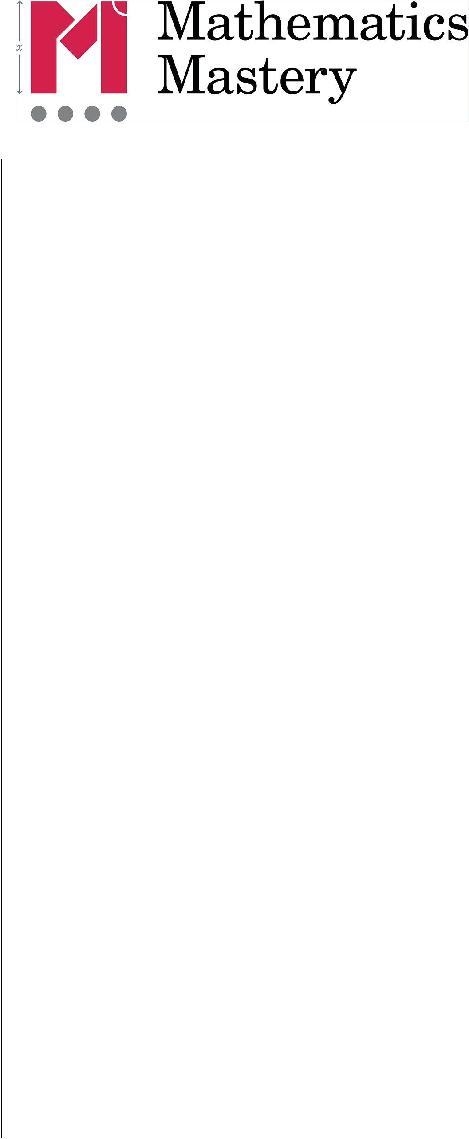


|  |  |  |  |
| --- | --- | --- | --- |
| **Strategy & guidance** |  | **CPA** |  |
|  |  | |  |
| **Written column method for calculations that** | As for the mental strategies, pupils should | |  |
| **require regrouping with up to 4-digits** | be exposed to concrete manipulatives | |  |
|  |  |
| *Dienes blocks should be used alongside the pictorial* | modelling the written calculations and | |  |
|  |  |  |
| *representations during direct teaching and can be* | should be able to represent their written | |  |
| *used by pupils both for support and challenge. Place* | work pictorially or with concrete | |  |
| *value counters can also be introduced at this stage.* | manipulatives when required. | |  |
| *This work revises and reinforces ideas from Key* | Again, they should be encouraged to | |  |
|  |  |  |
| *Stage 1, including the focus on place value –see Year* | calculate with known and derived facts and | |  |
| *2 exemplification.* | should not rely on counting images or | |  |
| *Direct teaching of the columnar method should* | manipulatives. |  |  |
|  |  |  |
| *require at least one element of regrouping, so that* |  |  |  |
| *pupils are clear about when it is most useful to use it.* |  |  |  |
| *Asking them ‘Can you think of a more efficient* |  |  |  |
| *method?’ will challenge hem to apply their number* |  |  |  |
| *sense / number facts to use efficient mental methods* |  |  |  |
| *where possible.* |  |  |  |
| *As in Year 2, pupils should be given plenty of practice* |  |  |  |
| *with calculations that require multiple separate* |  |  |  |
| *instances of regrouping. In Year 3 they become more* |  |  |  |
| *familiar with calculations that require ‘regrouping to* |  |  |  |
| *regroup’. Understanding must be secured through the* | 5 + 6 = 11 so I will have 11 ones which I | |  |
| *considered use of manipulatives and images,* | regroup for 1 ten and 1 one. | |  |
| *combined with careful use of language.* |  |  |  |
| *Pupils should be challenged as to whether this is the* | Regrouping (including multiple separate | |  |
| instances) |  |  |
| *most efficient method, considering whether mental* |  |  |
|  |  |  |
| *methods (such as counting on, using known number* | 672 + 136 | 734 –82 |  |
| *facts, round and adjust etc.) may be likelier to* |  |
|  |  |  |
| *produce an accurate solution.* | 468 + 67 | 831 - 76 |  |
|  |  |
| *Pupils requiring support might develop their* | 275 + 386 | 435 –188 |  |
| *confidence in the written method using numbers that* |  |
|  |  |  |
| *require no regrouping.* |  |  |  |
| *See Unit materials for extra guidance on this* | ‘Regrouping to regroup’ | |  |
|  |  |  |
| *strategy.* | 204 –137 |  |  |
|  |  |  |
|  | 1035 - 851 |  |  |
|  |  |  |  |



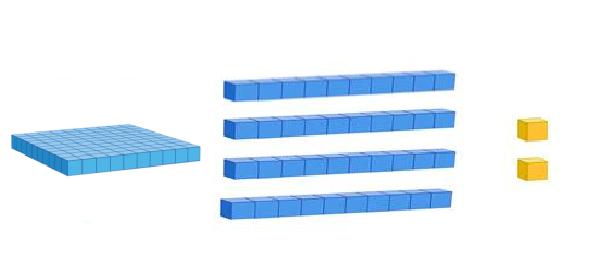
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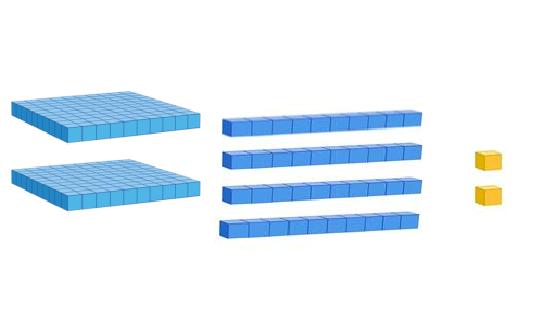


|  |  |
| --- | --- |
| **Strategy & guidance** | **CPA** |
|  |  |
| **Find 10, 100 more or less than a given number** | 142 + 100 = 242 |

*As pupils become familiar with numbers up to 1000, place value should be emphasised and comparisons drawn between adding tens, hundreds (and, in the last unit of the Summer term, thousands), including use of concrete manipulatives and appropriate images.*



*After initial teaching, this should be incorporated into transition activities and practised regularly.*



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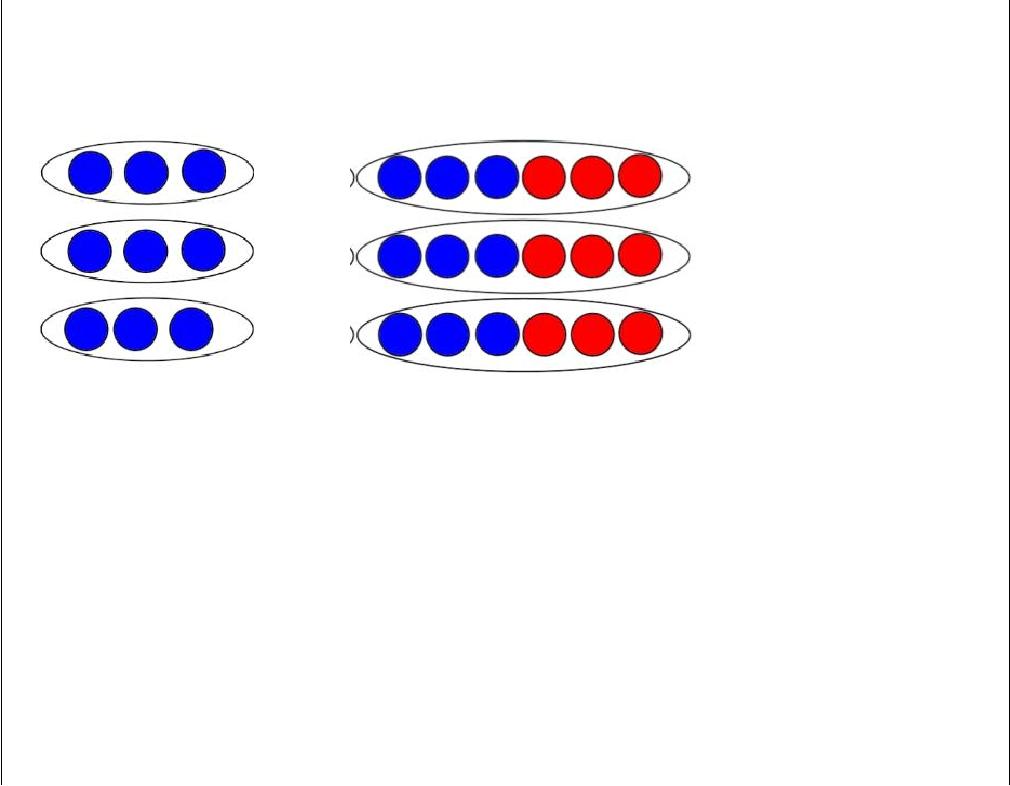
**National Curriculum objectives linked to multiplication and division**

**These objectives are explicitly covered through the strategies outlined in this document:**

* count from 0 in multiples of 4, 8, 50 and 100
* recall and use multiplication and division facts for the 3, 4, 6, and 8 multiplication tables
* write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental methods
* solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which *n* objects are connected to *m* objects

**Teachers should refer to definitions and guidance on the**  [**structures for multiplicatio**](#page8)**n**  [**and division**](#page8) **to provide a range of appropriate real-life contexts for calculations.**

|  |  |  |
| --- | --- | --- |
|  | **Y3 Multiplication** | |
|  |  |  |
| **Strategy & guidance** |  | **CPA** |
|  |  |  |
| **Doubling to derive** | 3 x 3 = 9 | 3 x 6 = double 9 = 18 |
| **new multiplication** |  |  |
| **facts** |  |  |

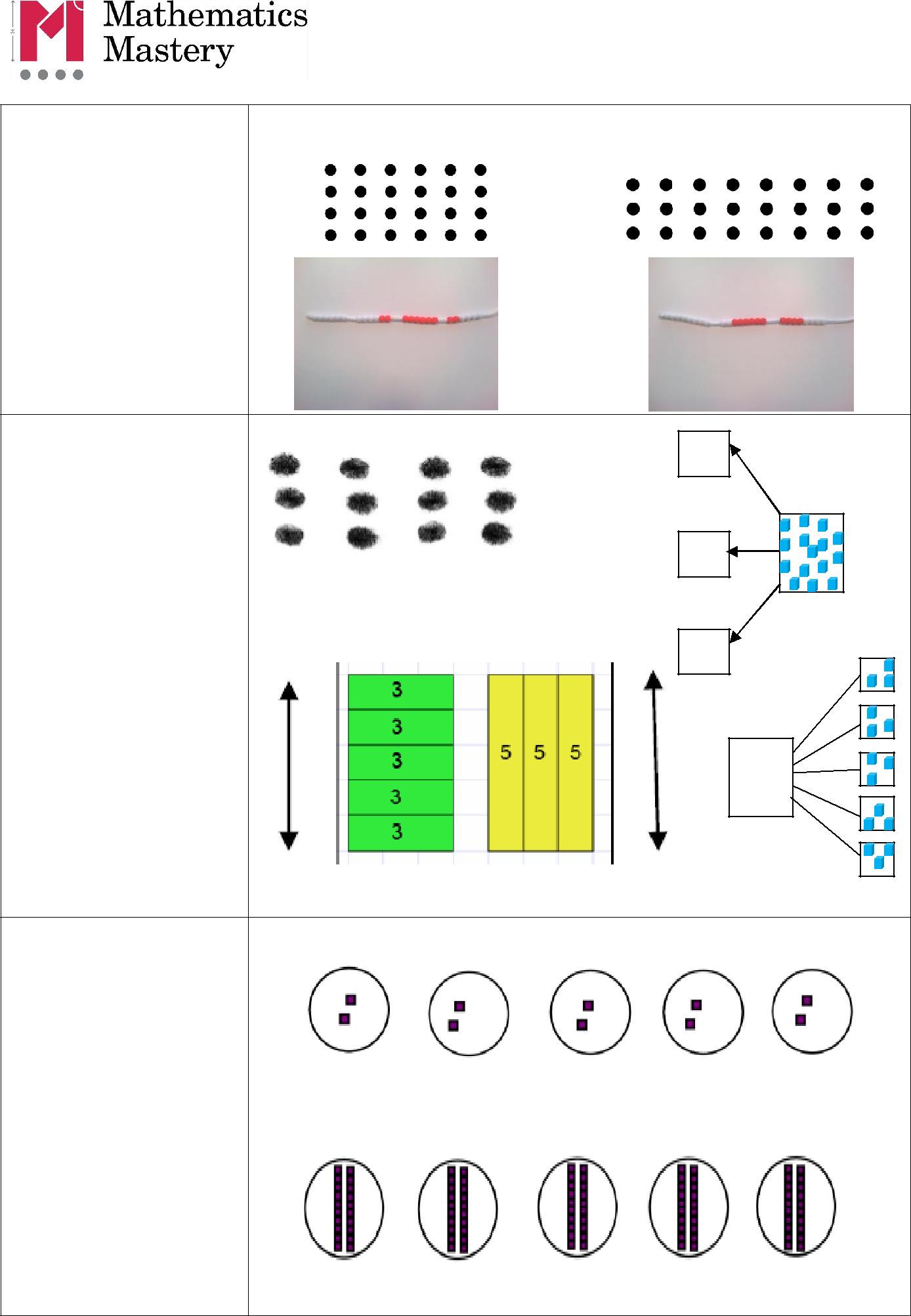


*Pupils continue to make use of the idea that facts from easier times tables can be used to derive facts from related times tables using doubling as a strategy.*

*This builds on the doubling strategy from Year 2.*

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|  |  |
| --- | --- |
| **Strategy & guidance** | **CPA** |
|  |  |
| **Skip counting in** |  |
| **multiples of 2, 3, 4, 5,** |  |
| **6, 8 and 10** |  |

*Rehearsal of previously learnt tables as well as new content for Year 3 should be incorporated into transition activities and practised regularly.*

**Use of part-part-whole model with arrays and bar models to establish commutativity and inverse relationship between multiplication and division**

*In these contexts pupils are able to identify all the equations in a fact family.*

***Also with digits***

**Ten times greater**

*Pupils’ work on this must be firmly based on concrete representations*

*–the language of ten times greater must be well modelled and understood to prevent the numerical*

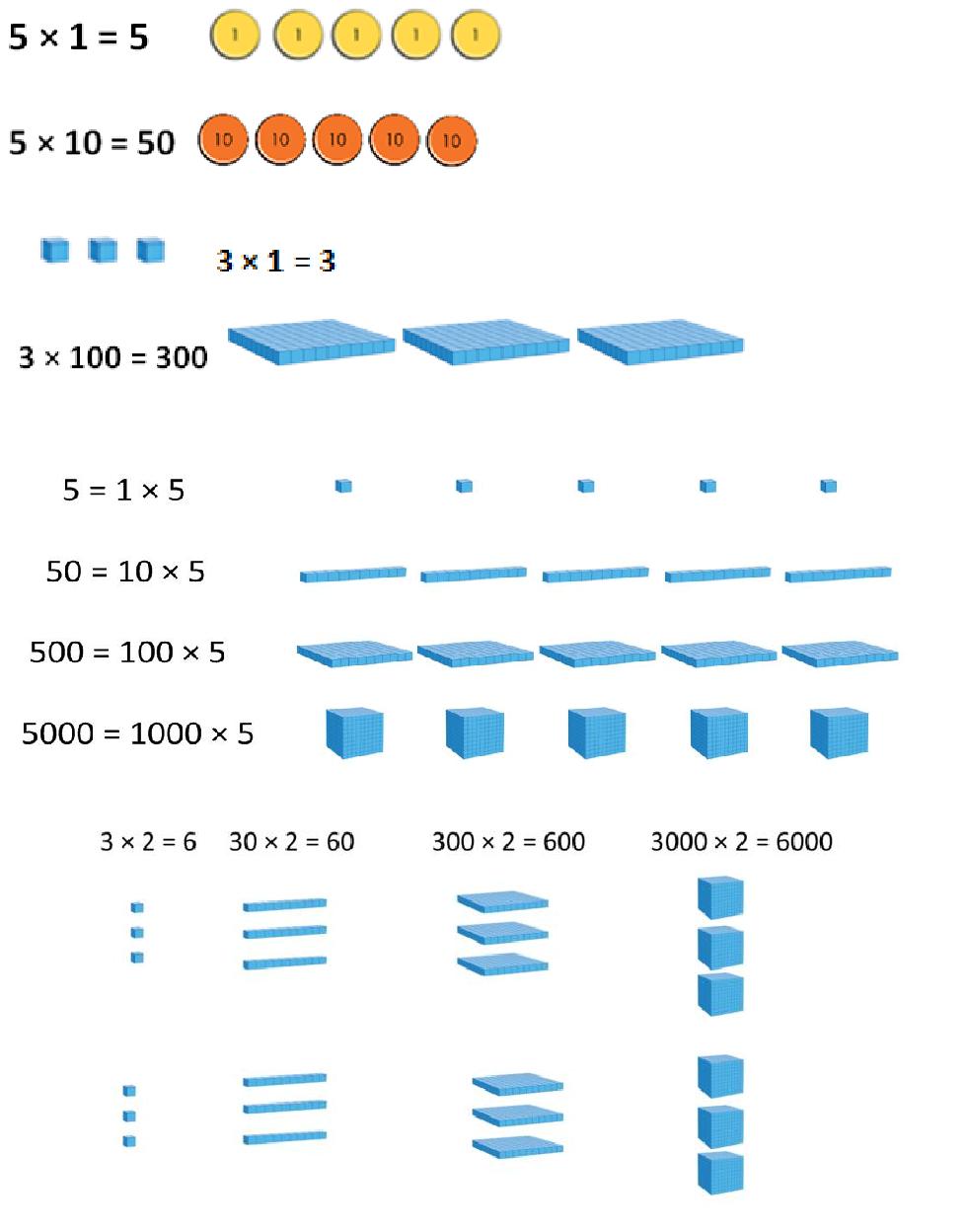
*Misconception of ‘adding a zero’.*

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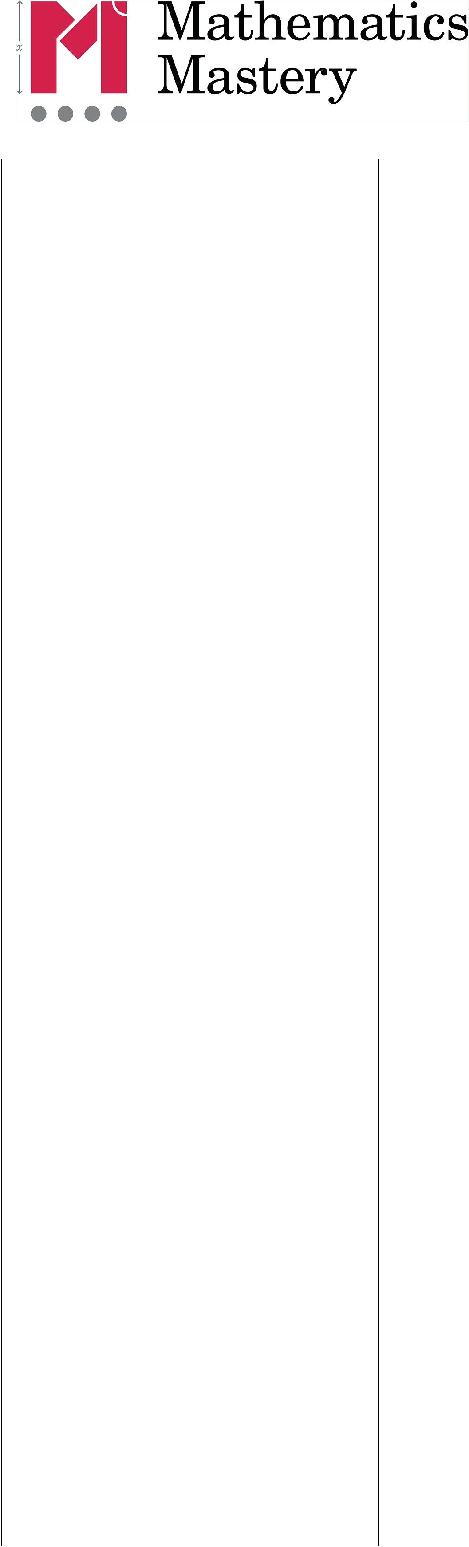


|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Strategy & guidance** |  | **CPA** | | | |  |
|  |  |  |  |  |  |  |
| **Multiplying by 10 and** |  |  |  |  |  |  |
| **100** |  |  |  |  |  |  |
| *Building on the ten times* |  |  |  |  |  |  |
| *greater work, pupils use* |  |  |  |  |  |  |
| *appropriate Dienes* |  |  |  |  |  |  |
| *blocks and place value* |  |  |  |  |  |  |
| *counters to multiply 2, 3,* |  |  |  |  |  |  |
| *4, 5 and 10 by 10, 100* |  |  |  |  |  |  |
| *and 1000.* |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| **Using known facts for** |  |  |  |  |  |  |
| **multiplying by** |  |  |  |  |  |  |
| **multiples of 10 and** |  |  |  |  |  |  |
| **100** |  |  |  |  |  |  |
| *Pupils’ growing* |  |  |  |  |  |  |
| *understanding of place* |  |  |  |  |  |  |
| *value, allows them to* |  |  |  |  |  |  |
|  |  |  |  |  |  |
| *make use of known facts* |  |  |  |  |  |  |
| *to derive multiplications* |  |  |  |  |  |  |
|  |  |  |  |  |  |
| *using powers of 10.* |  |  |  |  |  |  |
| *It is important to use* |  |  |  |  |  |  |
| *tables with which they* |  |  |  |  |  |  |
| *are already familiar (i.e.* |  |  |  |  |  |  |
| *not 7 or 9 tables in Year* |  |  |  |  |  |  |
| *3)* |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
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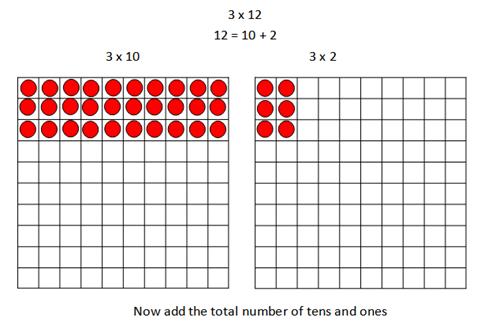


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|  |  |
| --- | --- |
| **Strategy & guidance** | **CPA** |
|  |  |
| **Multiplication of 2-** |  |
| **digit numbers with** |  |
| **partitioning (no** |  |
| **regrouping)** |  |



*Children should always consider whether partitioning is the best strategy –if it is possible to use strategies such as doubling (some may use doubling twice for ×4), they need to choose the most efficient strategy.*

*Children may wish to make jottings, including a full grid as exemplified here –but grid method is not a formal method and its only purpose is to record mental calculations. This supports the development of the*



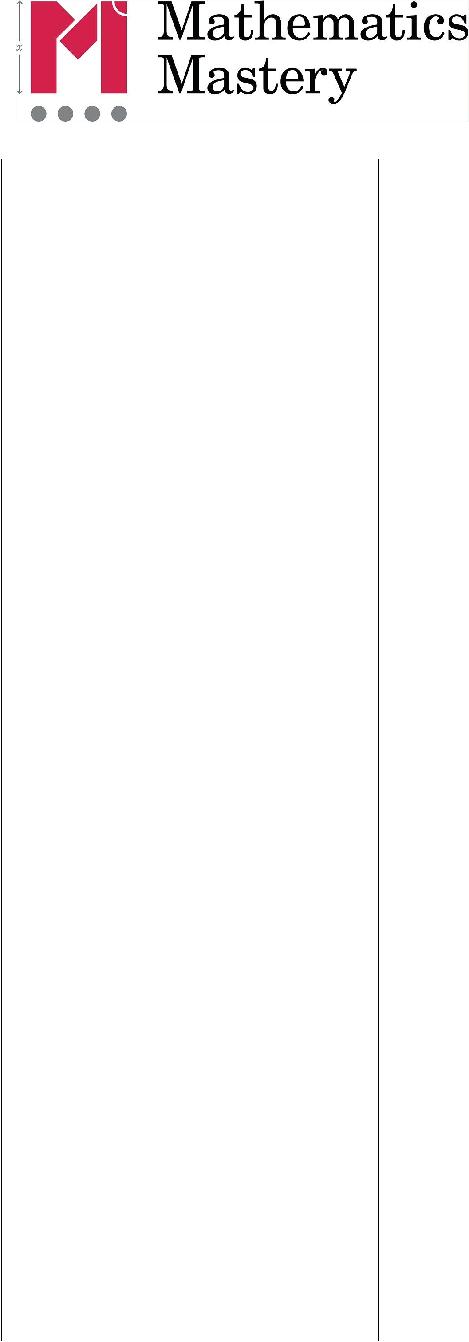
*necessary mental*  **3 x 12 = 36**

*calculating skills but does*

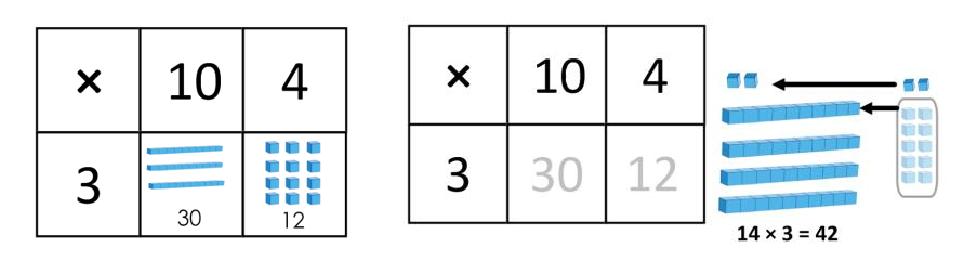
*not hinder the introduction of formal written methods in Year 4. Concrete manipulatives are essential to develop understanding.*

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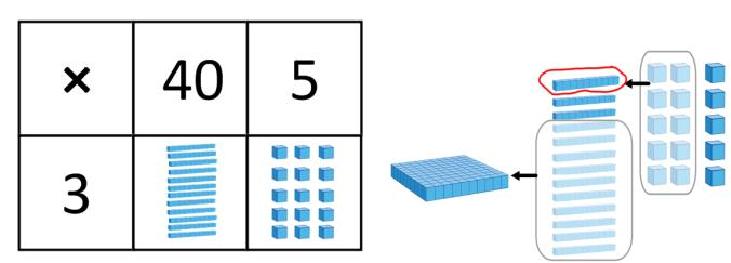
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|  |  |
| --- | --- |
| **Strategy & guidance** | **CPA** |
|  |  |
| **Multiplication of 2-** |  |
| **digit numbers with** |  |
| **partitioning** |  |
| **(regrouping)** |  |



*Using concrete manipulatives and later moving to using images that represent them, supports pupils’ early understanding, leading towards formal written methods in Year 4.*



*Once again, this is a mental strategy, which they may choose to support with informal jottings, including a full grid, as exemplified here.*

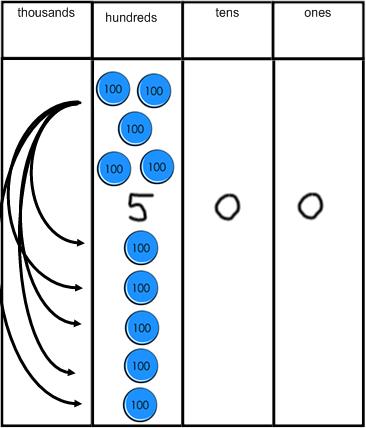
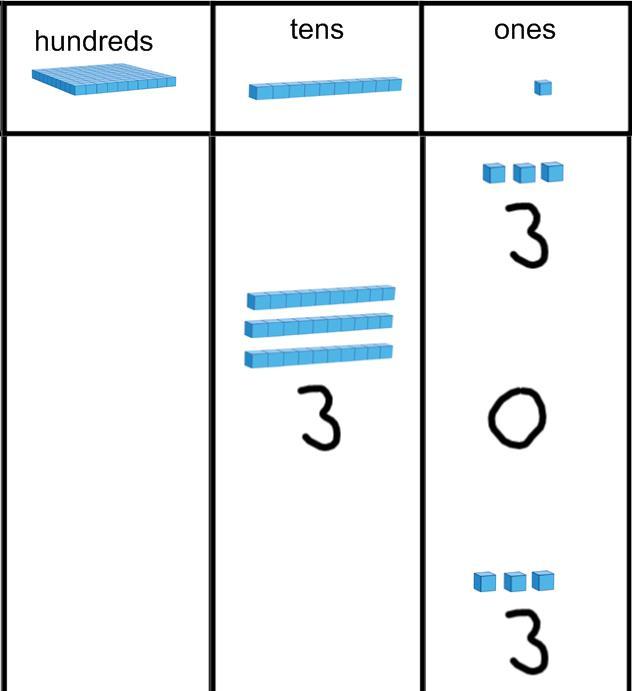
*Pupils must be encouraged to make use of their known multiplication facts and their knowledge of place value to calculate, rather than counting manipulatives.*

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|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | **Y3 Division** | |  |
|  |  |  |  |  |
| **Strategy & Guidance** |  |  | **CPA** |  |
|  |  |  |  |  |
| **Dividing multiples of** |  |  |  |  |
| **10, 100 and 1000 by** |  |  |  |  |
| **10, 100 and 1000** |  |  |  |  |
| **using scaling down** |  |  |  |  |
| *Pupils use the strategy of* |  |  |  |  |
| *‘scaling down’* |  |  |  |  |
| *representing numbers* |  |  |  |  |
| *with concrete* | **3 × 10 = 30** | | |  |
| *manipulatives and* |  |  |  |  |
| *making the value ten* |  |  |  |  |
| *times smaller.* |  |  |  |  |
|  | **30 ÷ 10 = 3** | | |  |
|  |  |  |  |  |
| **Dividing multiples of** |  |  |  |  |
| **10, 100 and 1000 by** |  |  |  |  |
| **10, 100 and 1000** | 500 ÷ 100 = |  |  |  |
| **using grouping** |  |  |  |  |
| My whole is 500 and the value of the | | |  |
| *Pupils divide by 10, 100* |  |
| equal parts is 100. How many parts are | | |  |
| *and 1000 by making* |  |
| there? | | |  |
| *groups of the divisor.* |  |
|  |  |  |  |
|  |  |  |  |  |



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Progression in calculations Year 4

**National curriculum objectives linked to addition and subtraction**

**These objectives are explicitly covered through the strategies outlined in this document:**

* add and subtract numbers with up to four digits, using the formal written methods of columnar addition and subtraction where appropriate
* find 1000 more or less than a given number
* estimate and use inverse operations to check answers to a calculation

N.B. There is no explicit reference to mental calculation strategies in the programmes of study for Year 4 in the national curriculum. However, with an overall aim for fluency, appropriate mental strategies should always be considered before resorting to formal written procedures, with the emphasis on pupils making their own choices from an increasingly sophisticated range of strategies.

**The following objectives should be planned for lessons where new strategies are being introduced and developed:**

* solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why
* solve simple measure and money problems involving fractions and decimals to two decimal places

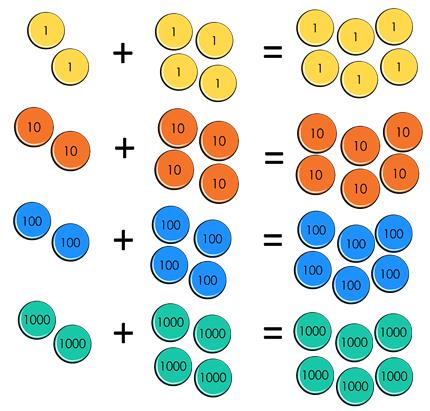
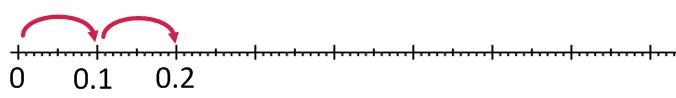
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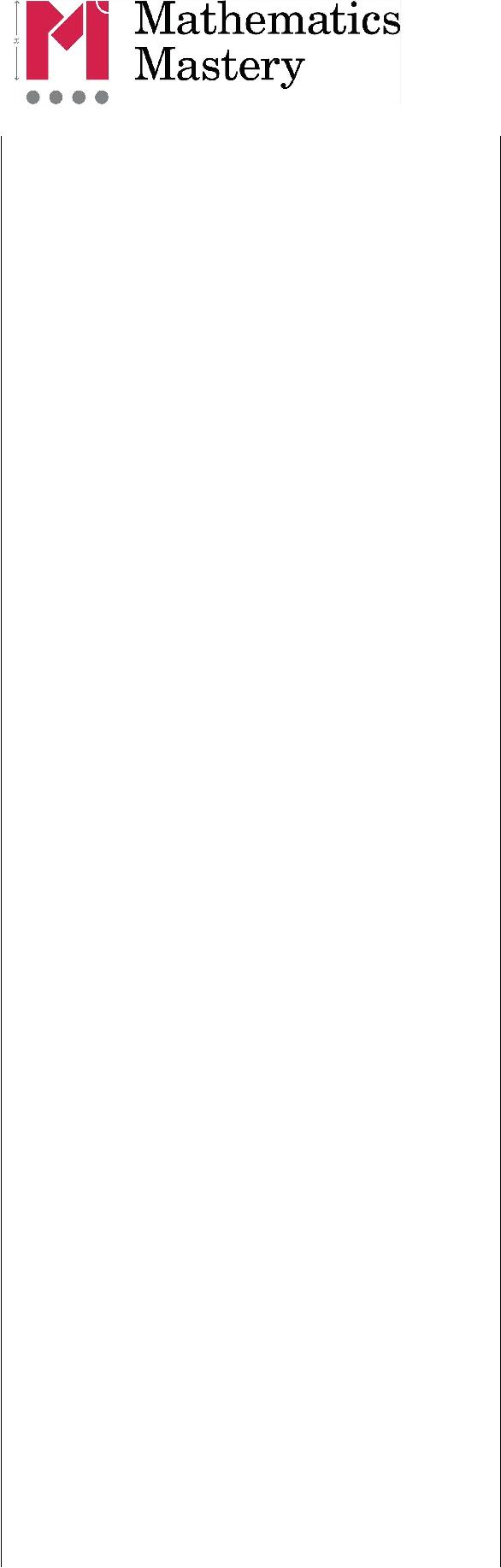
**Y4 Addition & Subtraction**

|  |  |  |
| --- | --- | --- |
| **Strategies & Guidance** | **CPA** |  |
|  |  |  |
| **Count forwards and backwards in** |  |  |
| **steps of 10, 100 and 1000 for any** |  |  |
| **number up to 10 000.** |  |  |
| *Pupils should count on and back in steps* |  |  |
| *of ten, one hundred and one thousand* |  |  |
| *from different starting points. These* |  |  |
| *should be practised regularly, ensuring* |  |  |
| *that boundaries where more than one* |  |  |
| *digit changes are included.* |  |  |
| **Count forwards and backwards in** | Pay particular attention to boundaries where regrouping |  |
| happens more than once and so more than one digit |  |
| **tenths and hundredths** |  |
| changes. |  |
|  |  |
|  | E.g. 990 + 10 or 19.9 + 0.1 |  |
|  |  |  |
| **Using known facts and knowledge** |  |  |
| **of place value to derive facts.** | 2 + 4 = 6 |  |
|  |  |
| **Add and subtract multiples of 10,** |  |  |
| **100 and 1000 mentally** | 20 + 40 = 60 |  |
| *Pupils extend this knowledge to mentally* |  |  |
| *adding and subtracting multiples of 10,* | 200 + 400 = 600 |  |
| *100 and 1000. Counting in different* |  |
|  |  |
| *multiples of 10, 100 and 1000 should be* |  |  |
| *incorporated into transition activities* | 2000 + 4000 = 6000 |  |
| *and practised regularly.* |  |  |
|  |  |  |
| **Adding and subtracting by** | See Y3 guidance on mental addition & subtraction, |  |
| **partitioning one number and** | remembering that use of concrete manipulatives and |  |
| **applying known facts.** |  |
| images in both teaching and reasoning activities will help |  |
|  |  |
| *By Year 4 pupils are confident in their* | to secure understanding and develop mastery. |  |
| *place value knowledge and are* |  |  |
| *calculating mentally both with* |  |  |
| *calculations that do not require* |  |  |
| *regrouping and with those that do.* |  |  |
|  |  |  |

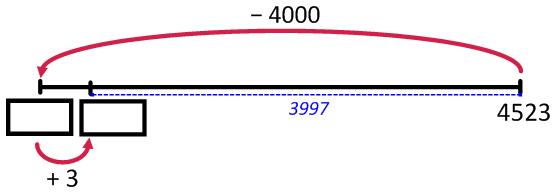
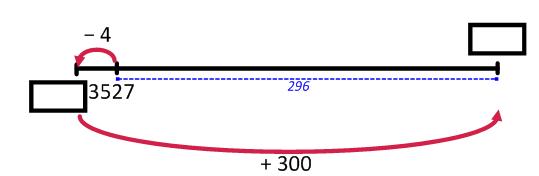
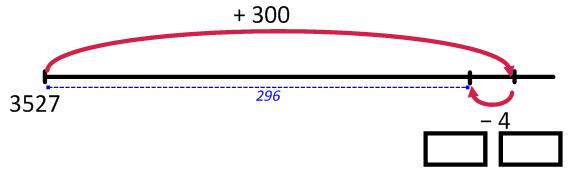


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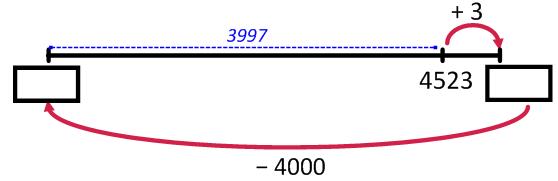
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| --- | --- | --- |
| **Strategies & Guidance** | **CPA** |  |
|  |  |  |
| **Round and adjust** | 3527 + 296 = 3827 - 4 |  |
| *Pupils should recognise that this* |  |  |
| *strategy is useful when adding and* |  |  |
| *subtracting near multiples of ten. They* |  |  |
| *should apply their knowledge of* |  |  |
| *rounding.* |  |  |
| *It is very easy to be confused about how* | Completing the same calculation but adjusting first: |  |
|  |  |
| *to adjust and so visual representations* | 3527 + 296 = 3523 + 300 |  |
| *and logical reasoning are essential to* |  |
|  |  |
| *success with this strategy.* |  |  |
| *Build flexibility by completing the same* |  |  |
| *calculation in a different order.* |  |  |
|  | 4523 –3997 = 523 + 3 |  |

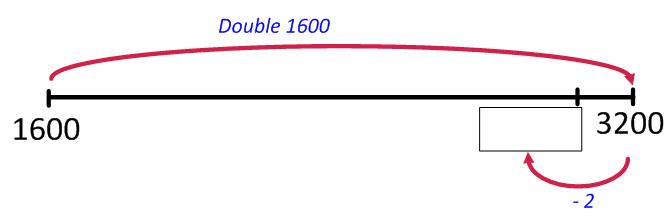


Completing the same calculation but adjusting first: 4523 –3997 = 4526 - 4000



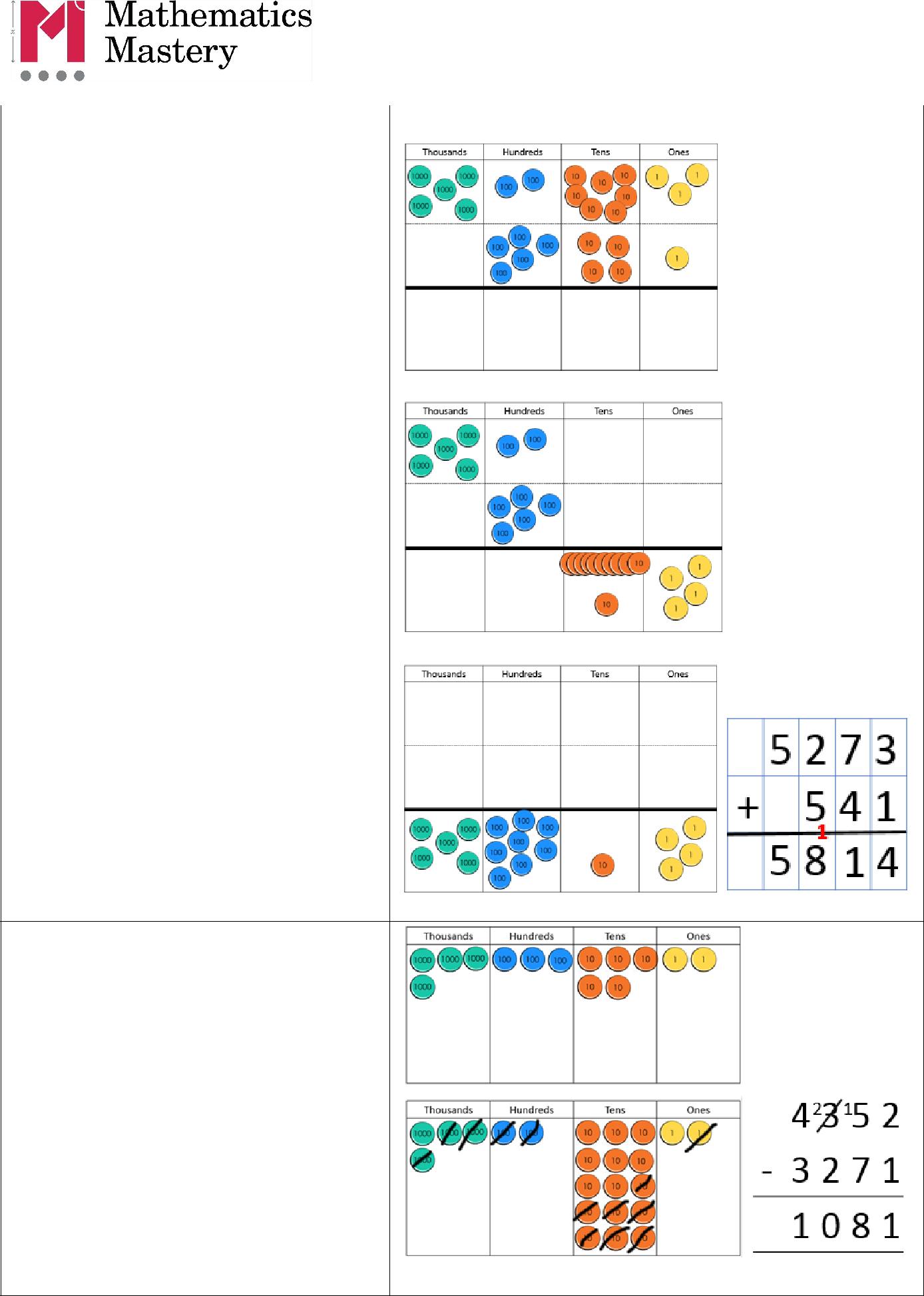
|  |  |
| --- | --- |
| **Near doubles** | 1600 + 1598 = double 1600 –2 |

*Pupils should be able to double numbers up to 100 and use this to derive doubles for multiples of ten. These facts can be adjusted to calculate near doubles.*



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|  |  |
| --- | --- |
| **Strategies & Guidance** | **CPA** |
|  |  |
| **Written column methods for** |  |
| **addition** |  |

*Place value counters are a useful manipulative for representing the steps of the formal written method. These should be used alongside the written layout to ensure conceptual understanding and as a tool for explaining.*

*This method and the language to use are best understood through the tutorial videos found*  [*here*](http://toolkit.mathematicsmastery.org/classroom-planning/classroom-resources) *on the toolkit.*

**Written column methods for subtraction**

*Place value counters are a useful manipulative for representing the steps of the formal written method. These should be used alongside the written layout to ensure conceptual understanding and as a tool for explaining.*

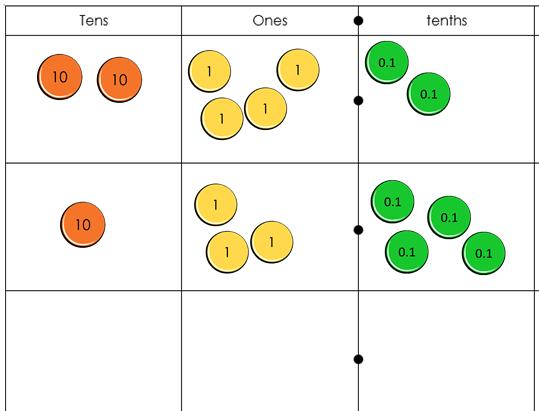
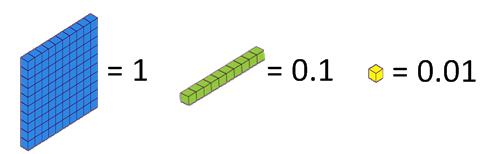
*This method and the language to use are best understood through the tutorial videos on the toolkit.*

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|  |  |
| --- | --- |
| **Strategies & Guidance** | **CPA** |
|  |  |
| **Calculating with decimal numbers** |  |
| *Assign different values to Dienes* |  |
| *equipment. If a Dienes 100 block has the* |  |
| *value of 1, then a tens rod has a value of* |  |
| *0.1 and a ones cube has a value of 0.01.* |  |
| *These can then be used to build a* |  |
| *conceptual understanding of the* | 24.2 + 13.4 = |
| *relationship between these.* |  |
| *Place value counters are another useful* |  |
| *manipulative for representing decimal* |  |
| *numbers.* |  |
| *All of the calculation strategies for* |  |
| *integers (whole numbers) can be used to* |  |
| *calculate with decimal numbers.* |  |
|  |  |



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**National Curriculum objectives linked to multiplication and division**

**These objectives are explicitly covered through the strategies outlined in this document:**

* count from 0 in multiples of 6, 7, 9, 25 and 1000
* recall and use multiplication and division facts for multiplication tables up to 12 × 12
* write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
* recognise and use factor pairs and commutativity in mental calculations
* use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
* multiply two-digit and three-digit numbers by a one-digit number using formal written layout
* find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths.

**The following objectives should be planned for lessons where new strategies are being introduced and developed:**

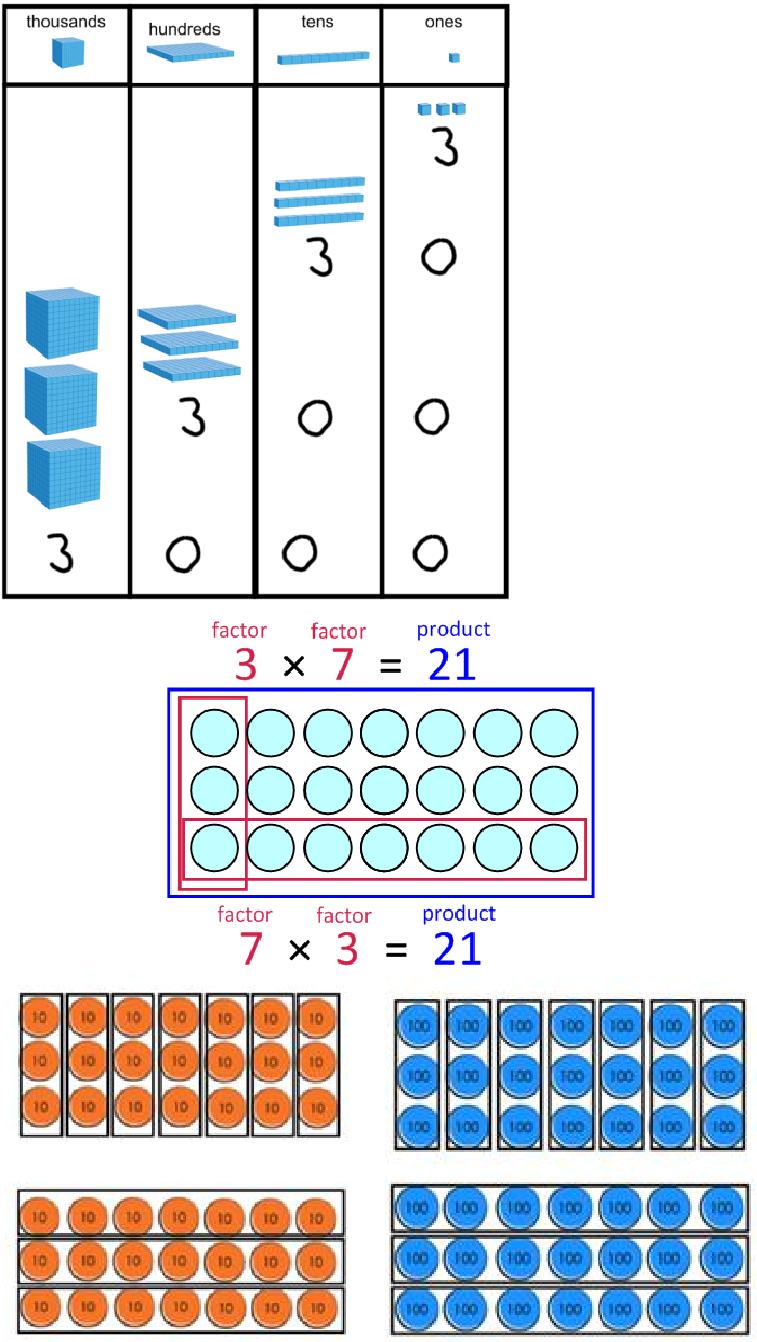
* solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as *n* objects are connected to *m* objects.

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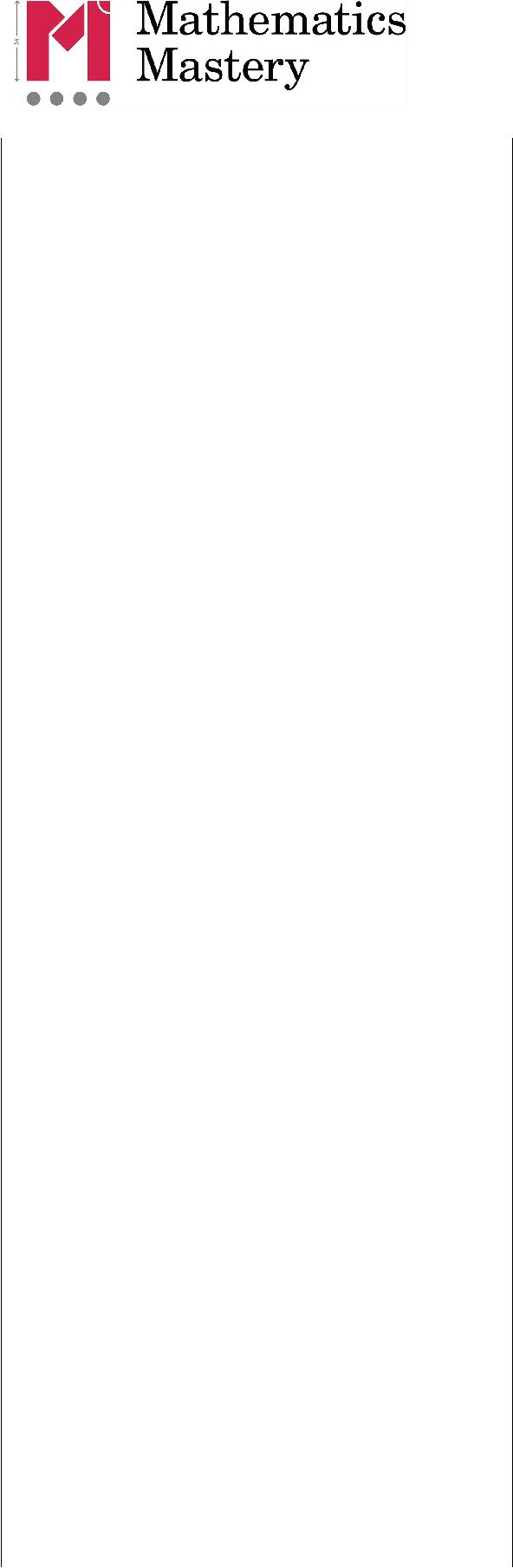


|  |  |  |  |
| --- | --- | --- | --- |
| **Y4 Multiplication** | |  |  |
|  |  |  |  |
| **Strategies & Guidance** |  | **CPA** |  |
|  |  |  |  |
| **Multiplying by 10 and 100** |  |  |  |
| *When you multiply by ten, each part is* |  |  |  |
| *ten times greater. The ones become tens,* |  |  |  |
| *the tens become hundreds, etc.* |  |  |  |
| *When multiplying whole numbers, a zero* |  |  |  |
| *holds a place so that each digit has a* |  |  |  |
| *value that is ten times greater.* |  | 3 x 10 = 30 |  |
|  |  |  |
| *Repeated multiplication by ten will build* |  |  |  |
| *an understanding of multiplying by 100* |  |  |  |
| *and 1000* |  | 3 x 100 = 300 |  |
|  |  |  |
|  |  | 3 x 1000 = 3000 |  |
|  |  |  |  |
| **Using known facts and place value** |  |  |  |
| **for mental multiplication involving** |  |  |  |
| **multiples of 10 and 100** |  |  |  |
| *Pupils use their growing knowledge of* |  |  |  |
| *multiplication facts, place value and* |  |  |  |
| *derived facts to multiply mentally.* |  |  |  |
| *Emphasis is placed on understanding the* |  |  |  |
| *relationship (10 times or 100 times* |  |  |  |
| *greater) between a known number fact* |  |  |  |
| *and one to be derived, allowing far* |  |  |  |
| *larger ‘fact families’ to be derived from a* |  |  |  |
| *single known number fact.* |  |  |  |
| *Knowledge of commutativity (that* |  |  |  |
| *multiplication can be completed in any* |  |  |  |
| *order) is used to find a range of related* |  |  |  |
| *facts.* |  |  |  |
|  | 30 x 7 = 210 | 300 x 7 = 2100 |  |
|  | 70 x 3 = 210 | 700 x 3 = 2100 |  |
|  | 7 x 30 = 210 | 7 x 300 = 2100 |  |
|  | 3 x 70 = 210 | 3 x 700 = 2100 |  |
|  |  |  |  |

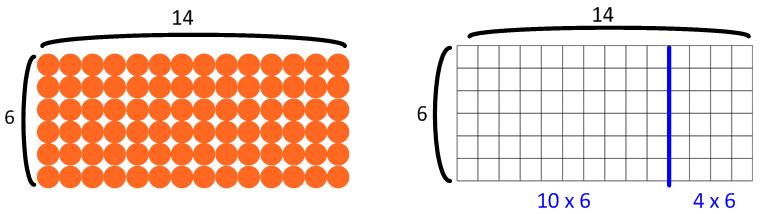


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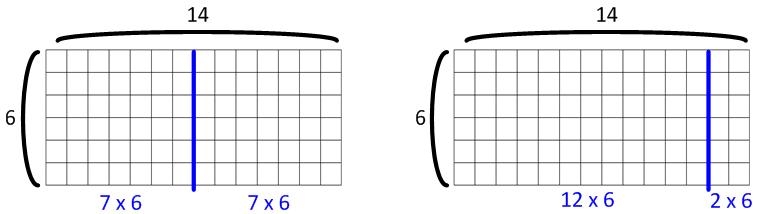
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|  |  |
| --- | --- |
| **Strategies & Guidance** | **CPA** |
|  |  |
| **Multiplying by partitioning one** | 14 x 6 |
| **number and multiplying each part** |  |



*Pupils build on mental multiplication strategies and develop an explicit understanding of distributive law, which allows them to explore new strategies to make more efficient calculations.*

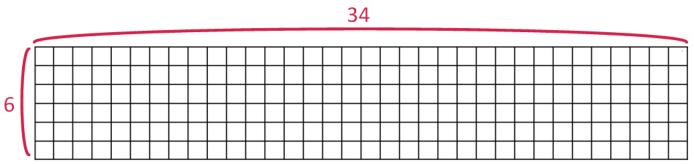


*As well as partitioning into tens and ones (a familiar strategy), they begin to explore compensating strategies and factorisation to find the most efficient solution to a calculation.*

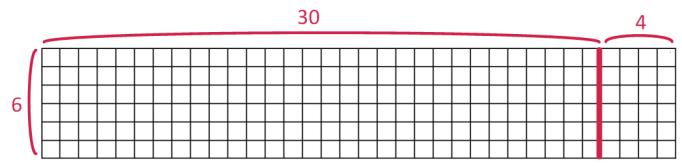
**Distributive law**

34 × 6

**a x (b + c) = a x b + a x c**



30 × 6 + 4 × 6



|  |  |  |
| --- | --- | --- |
| **Mental multiplication of three 1-** | Four pots each containing two flowers which each have |  |
| **digit numbers, using the associative** | seven petals. How many petals in total? |  |
| **law** |  |
|  |  |
| *Pupils first learn that multiplication can* |  |  |
| *be performed in any order, before* |  |  |
| *applying this to choose the most efficient* |  |  |
| *order to complete calculations, based on* |  |  |
| *their increasingly sophisticated number* |  |  |
| *facts and place value knowledge.* | (4 x 2) x 7 or 4 x (2 x 7) |  |
|  |  |

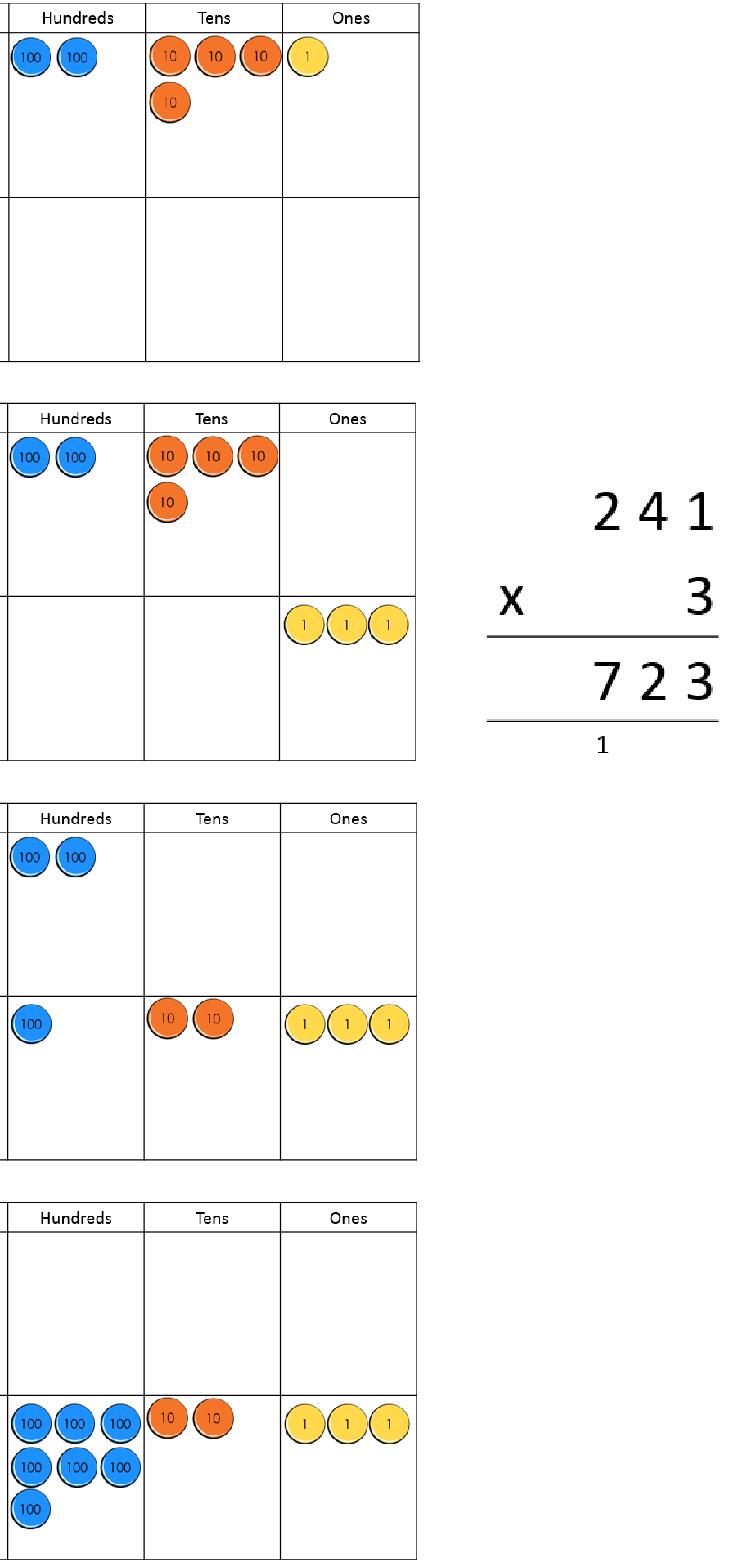


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| --- | --- | --- |
| **Strategies & Guidance** | **CPA** |  |
|  |  |  |
| **Short multiplication of 3-digit** | Exemplification of this process is best understood |  |
| **number by 1-digit number** | through viewing the video tutorial |  |
|  |  |
| *To begin with pupils are presented with* |  |  |
| *calculations that require no regrouping* | To calculate 241 x 3, |  |
| *or only regrouping from the ones to the* | represent the |  |
| *tens. Their conceptual understanding is* | number 241. |  |
| *supported by the use of place value* |  |
| Multiply each part |  |
| *counters, both during teacher* |  |
| by 3, regrouping as |  |
| *demonstrations and during their own* |  |
| *practice.* | needed. |  |
| *With practice pupils will be able to* |  |  |
| *regroup in any column, including from* |  |  |
| *the hundreds to the thousands, including* |  |  |
| *being able to multiply numbers* |  |  |
| *containing zero and regrouping through* |  |  |
| *multiple columns in a single calculation.* |  |  |
| *This method and the language to use are* |  |  |
| *best understood through the tutorial* |  |  |
| *videos found on the toolkit.* |  |  |
|  |  |  |

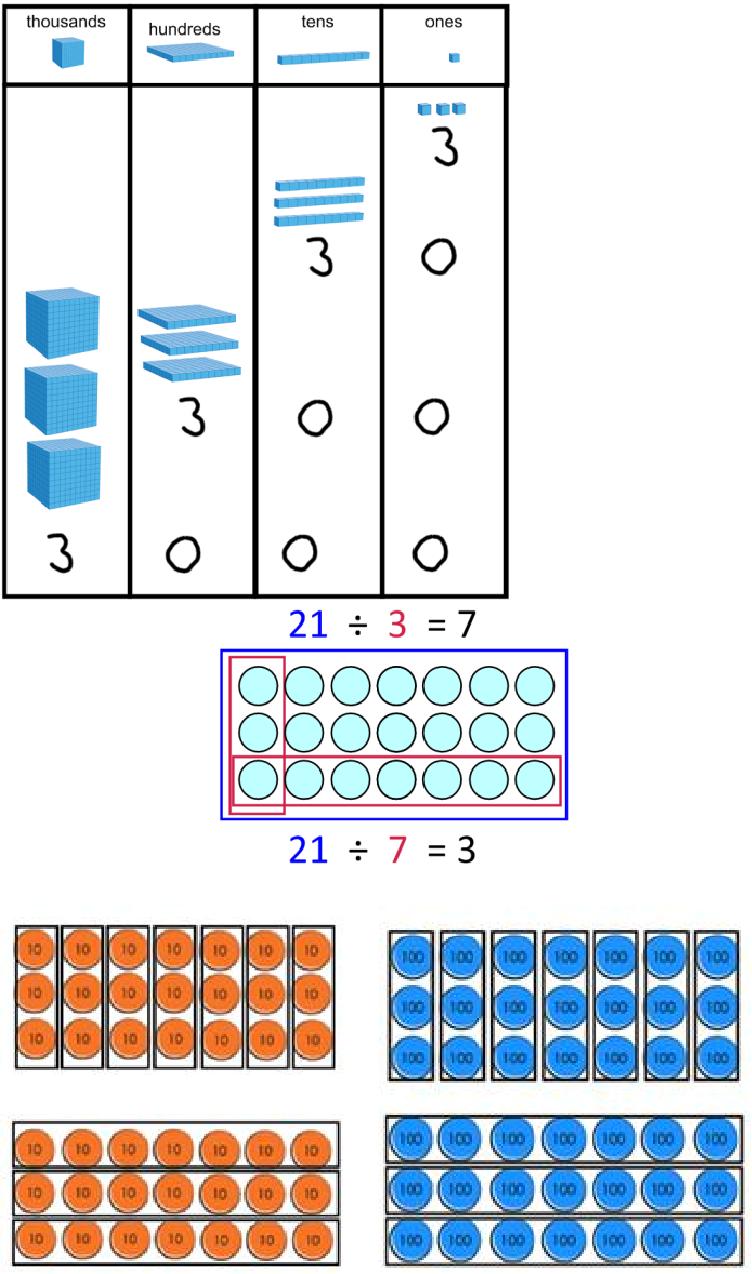


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|  |  |  |  |
| --- | --- | --- | --- |
|  | **Y4 Division** |  |  |
|  |  |  |  |
| **Strategies & Guidance** |  | **CPA** |  |
|  |  |  |  |
| **Dividing by 10 and 100** |  |  |  |
| *When you divide by ten, each part is* |  | 30 ÷ 10 = 3 |  |
| *ten times smaller. The hundreds* |  |  |
|  | 300 ÷ 100 = 3 |  |
| *become tens and the tens become ones.* |  |  |
|  | 3000 ÷ 1000 = 3 |  |
| *Each digit is in a place that gives it a* |  |  |
|  |  |  |
| *value that is ten times smaller.* |  |  |  |
| *When dividing multiples of ten, a place* |  | 300 ÷ 10 = 30 |  |
| *holder is no longer needed so that each* |  | 3000 ÷ 100 = 30 |  |
| *digit has a value that is ten times* |  |  |  |
| *smaller. E.g. 210 ÷ 10 = 21* |  |  |  |
|  |  | 3000 ÷ 10 = 300 |  |
|  |  |  |  |
| **Derived facts** |  |  |  |
| *Pupils use their growing knowledge of* |  |  |  |
| *multiplication facts, place value and* |  |  |  |
| *derived facts to multiply mentally.* |  |  |  |
| *Understanding of the inverse* |  |  |  |
| *relationship between multiplication* |  |  |  |
| *and division allows corresponding* |  |  |  |
| *division facts to be derived.* |  |  |  |
|  | 210 ÷ 7 = 30 | 2100 ÷ 7 = 300 |  |
|  | 210 ÷ 3 = 70 | 2100 ÷ 3 = 700 |  |
|  | 210 ÷ 30 = 7 | 2100 ÷ 300 = 7 |  |
|  | 210 ÷ 70 = 3 | 2100 ÷ 700 = 3 |  |
|  |  |  |  |

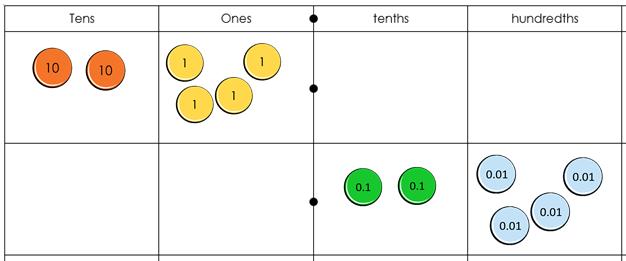
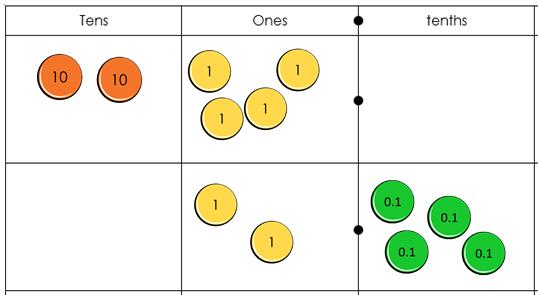
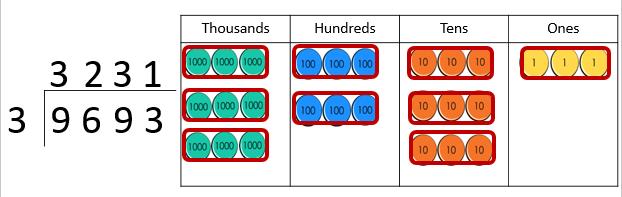


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| --- | --- | --- |
| **Strategies & Guidance** | **CPA** |  |
|  |  |  |
| **Short division of 4-digit numbers** | **Division as sharing** |  |
| **by 1-digit numbers** |  |  |
| *Pupils start with dividing 4-digit* |  |  |
| *numbers by 2, 3 and 4, where no* |  |  |
| *regrouping is required. Place value* |  |  |
| *counters are used simultaneously in a* |  |  |
| *place value chart, to develop conceptual* |  |  |
| *understanding.* |  |  |
| *They progress to calculations that* |  |  |
| *require regrouping in the hundreds or* |  |  |
| *tens columns.* | **Division as grouping** |  |
|  |  |
| *Pupils build on their conceptual* |  |  |
| *knowledge of division to become* |  |  |
| *confident with dividing numbers where* |  |  |
| *the tens digit is smaller than the* |  |  |
| *divisor, extending this to any digit* |  |  |
| *being smaller than the divisor.* |  |  |
| *Exemplification of this method and the* |  |  |
| *language to use are best understood* |  |  |
| *through viewing the tutorial videos* |  |  |
| *found on the toolkit.* |  |  |
|  |  |  |
| **Division of a one- or two-digit** | 24 ÷ 10 = 2.4 |  |
| **number by 10 and 100, identifying** |  |  |
| **the value of the digits in the** |  |  |
| **answer as ones, tenths and** |  |  |
| **hundredths** |  |  |
| *When you divide by ten, each part is* |  |  |
| *ten times smaller. The tens become ones* |  |  |
| *and the ones become tenths. Each digit* |  |  |
| *is in a place that gives it a value that is* |  |  |
| *ten times smaller.* |  |  |
|  | 24 ÷ 100 = 0.24 |  |
|  |  |  |



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Progression in calculations Year 5 + Year 6

**Year 5 and Year 6 are together because the calculation strategies used are broadly similar, with Year 6 using larger and smaller numbers. Any differences for Year 6 are highlighted in red.**

**National Curriculum objectives linked to integer addition and subtraction**

**These objectives are explicitly covered through the strategies outlined in this document:**

* add and subtract numbers mentally with increasingly large numbers
* add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
* use negative numbers in context, and calculate intervals across zero
* perform mental calculations, including with mixed operations and large numbers
* use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy

**The following objectives should be planned for lessons where new strategies are being introduced and developed:**

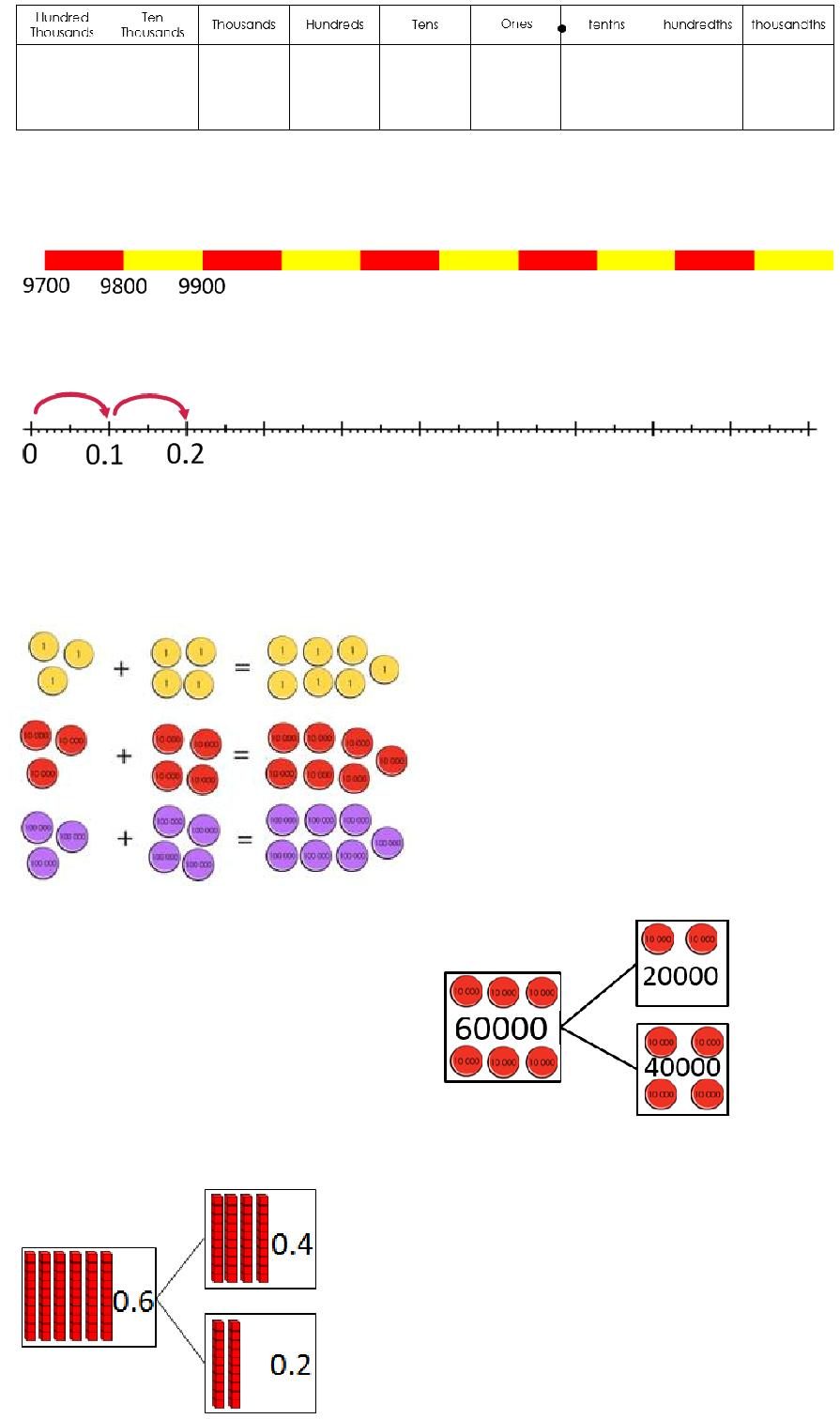
* use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
* solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
* solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign.

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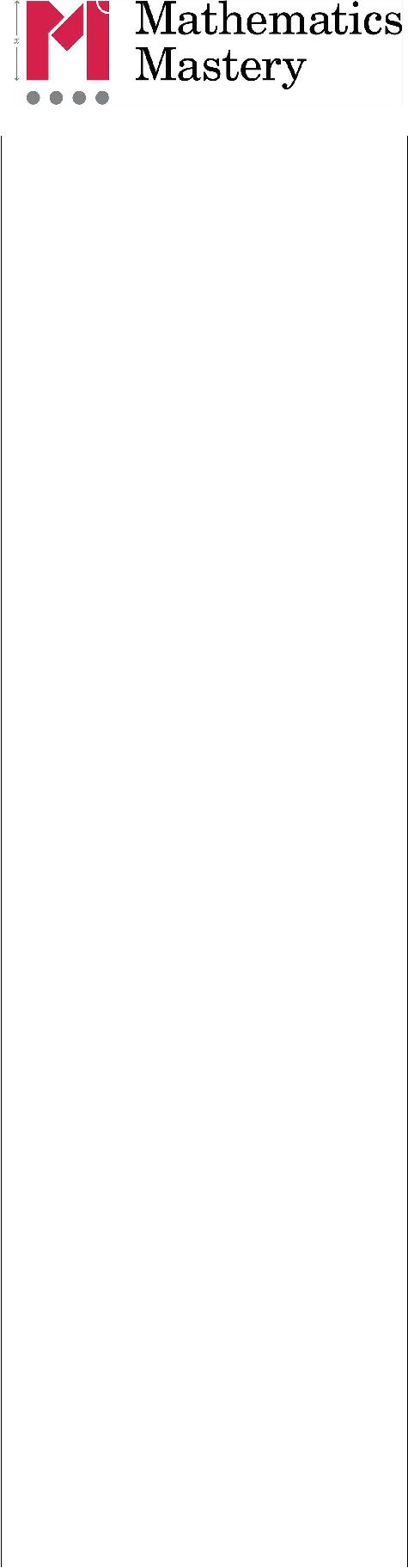


|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | **Y5 and Y6 Addition & Subtraction** | |  |
|  |  |  |  |  |
|  | **Strategies & Guidance** | | **CPA** |  |
|  | | |  |  |
| **Count forwards or** | | | Support with place value counters on a place value chart, |  |
| **backwards in steps of** | | | repeatedly adding the same counter and regrouping as needed. |  |
| **powers of 10 for any given** | | |  |
|  |  |
| **number up to 1 000 000** | | |  |  |
| *Skip counting forwards and* | | |  |  |
| *backwards in steps of powers of* | | |  |  |
| *10 (i.e. 10, 100, 1000, 10 000* | | |  |  |
| *and 100 000) should be* | | | Counting sticks and number lines: |  |
| *incorporated into transition* | | |  |  |
| *activities and practised* | | |  |  |
| *regularly.* | | |  |  |
| *In Year 5 pupils work with* | | |  |  |
| *numbers up to 1 000 000 as well* | | |  |  |
| *as tenths, hundredths and* | | |  |  |
| *thousandths.* | | | Pay particular attention to boundaries where regrouping |  |
|  |  |  |  |
|  | | |  |  |
| *In* | *Year 6* | *pupils work with* | happens more than once and so more than one-digit changes. |  |
| *numbers up to 10 000 000.* | | | e.g. 9900 + 100 = 10000 or 99 000 + 1000 = 100 000 |  |
|  |  |  |  |
|  | | |  |  |
| **Using known facts and** | | | 3 + 4 = 7 |  |
| **understanding of place** | | |  |  |
| **value to derive** | | | 30 000 + 40 000 = 70 000 |  |
| *Using the following language* | | | 300 000 + 400 000 = 700 000 |  |
| *makes the logic explicit: I know* | | |  |  |
| *three ones plus four ones is* | | |  |  |
| *equal to seven ones. Therefore,* | | |  |  |
| *three ten thousands plus four* | | |  |  |
| *ten thousands is equal to seven* | | | 20 000 + 40 000 = 60 000 |  |
| *ten thousands.* | | |  |
| 40 000 + 20 000 = 60 000 |  |
|  |  |  |  |
| *In Year 5 extend to multiples of* | | | 60 000 - 40 000 = 20 000 |  |
| *10 000 and 100 000 as well as* | | | 60 000 - 20 000 = 40 000 |  |
| *tenths, hundredths and* | | |  |  |
| *thousandths.* | | |  |  |
|  | | |  |  |
| *In* | *Year 6* | *extend to multiples of* | 0.6 = 0.2 + 0.4 |  |
| *one million.* | | |  |
|  |  |
|  |  |  | 0.6 = 0.4 + 0.2 |  |
| *These derived facts should be* | | | 0.2 = 0.6 - 0.4 |  |
| *used to estimate and check* | | | 0.4 = 0.6 - 0.2 |  |
| *answers to calculations.* | | |  |
|  |  |
|  |  | **Y6 Bar modelling also** |  |  |



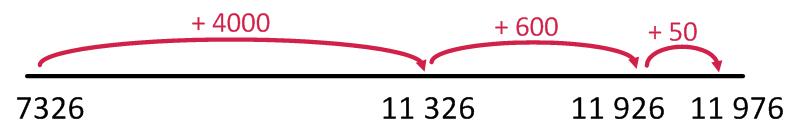
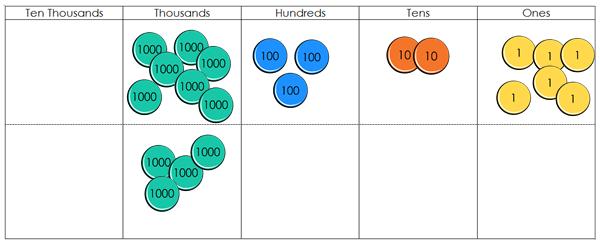
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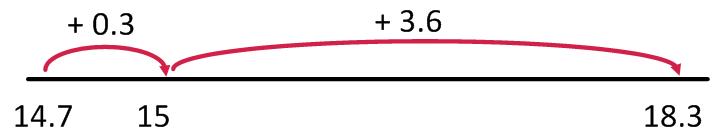


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| **Strategies & Guidance** | **CPA** |

|  |  |  |
| --- | --- | --- |
| **Partitioning one number** | **Partitioning into place value amounts (canonical partitioning):** |  |
| **and applying known facts to** |  |  |
| **add.** | 4650 + 7326 = 7326 + 4000 + 600 + 50 |  |
| *Pupils can use this strategy* |  |  |
| *mentally or with jottings as* |  |  |
| *needed.* |  |  |
| *Pupils should be aware of the* |  |  |
| *range of choices available when* |  |  |
| *deciding how to partition the* |  |  |
| *number that is to be added.* |  |  |
| *They should be encouraged to* | With place value counters, represent the larger number and then |  |
| *count on from the number of* | add each place value part of the other number. The image above |  |
| *greater value as this will be* |  |
| shows the thousands being added. |  |
| *more efficient. However, they* |  |
|  |  |
| *should have an understanding* | Represent pictorially with an empty numberline: |  |
| *of the commutative law of* |  |
|  |  |
| *addition, that the parts can be* |  |  |
| *added in any order.* |  |  |
| *Pupils have experience with* |  |  |
| *these strategies with smaller* | **Partitioning in different ways (non-canonical partitioning):** |  |
| *numbers from previous years* |  |
|  |  |
| *and so the focus should be on* | Extend the ‘Make ten’ strategy (see guidance in Y1 or Y2) to |  |
| *developing flexibility and* |  |
| count on to a multiple of 10. |  |
| *exploring efficiency.* |  |
|  | 6785 + 2325 = 6785 + 15 + 200 + 2110 |  |



The strategy can be used with decimal numbers, Make one: 14.7 + 3.6 = 14.7 + 0.3 + 3.3 = 15 + 3.3



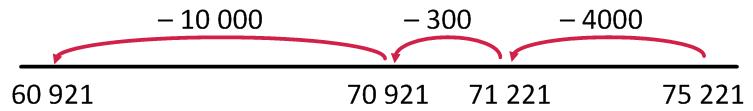
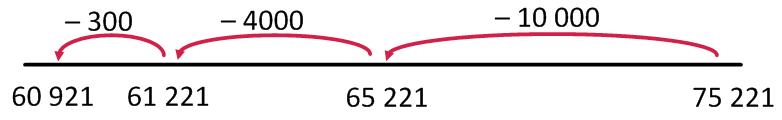
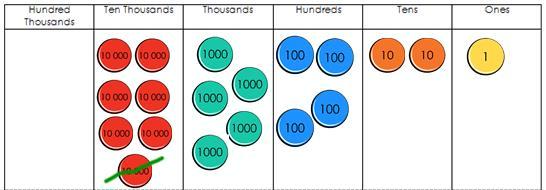
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| **Strategies & Guidance** | **CPA** |

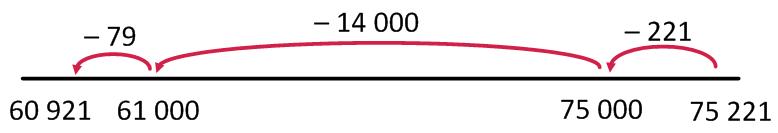
|  |  |  |
| --- | --- | --- |
| **Subtraction by partitioning** | **Partitioning into place value amounts (canonical partitioning):** |  |
| **and applying known facts.** |  |  |
| *Pupils can use this strategy* | 75 221 –14 300 = 75 221 –10 000 –4000 –300 |  |
|  |  |
| *mentally or with jottings as* |  |  |
| *needed.* |  |  |
| *Pupils should be aware of the* |  |  |
| *range of choices available when* |  |  |
| *deciding how to partition the* |  |  |
| *number that is to be subtracted.* |  |  |
| *Pupils have experience with* |  |  |
| *these strategies with smaller* | Represent pictorially with a number line, starting on the right and |  |
| *numbers from previous years* | having the arrows jump to the left: |  |
| *and so the focus should be on* |  |  |
| *developing flexibility and* |  |  |
| *exploring efficiency.* |  |  |
|  | Develop understanding that the parts can be subtracted in any |  |
|  | order and the result will be the same: |  |



**Partitioning in different ways (non-canonical partitioning):**

Extend the ‘Make ten’ strategy (see guidance in Y1 or Y2) to

count back to a multiple of 10.



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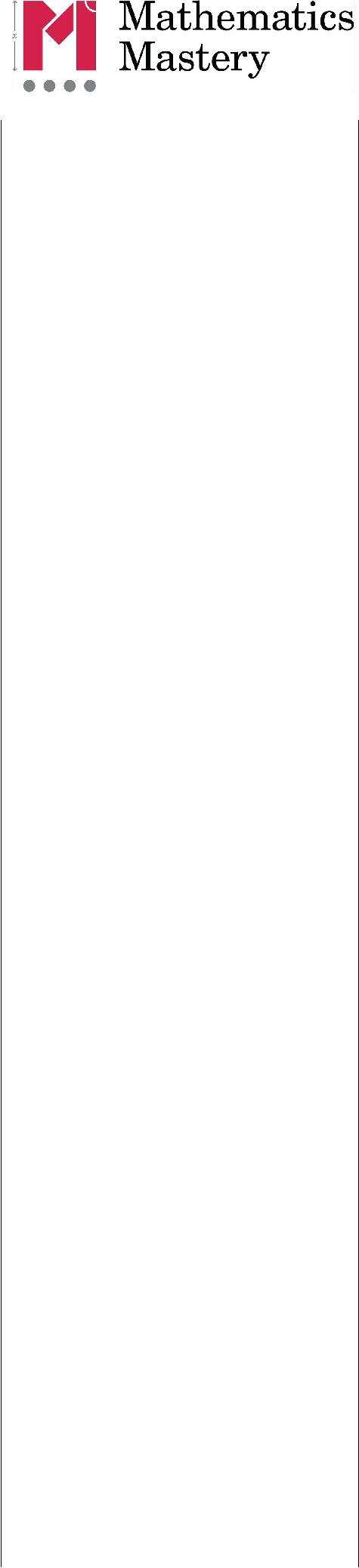
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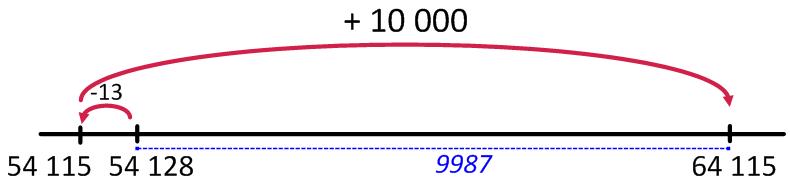
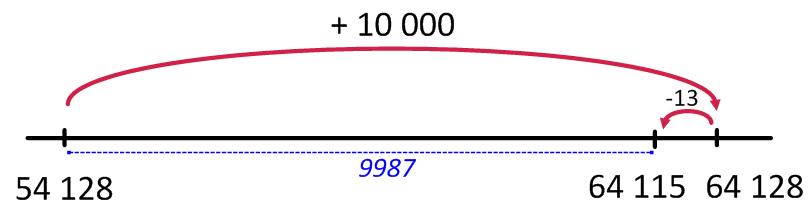
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| --- | --- | --- |
| **Strategies & Guidance** | **CPA** |  |
|  |  |  |
| **Calculate difference by** | 75 221 –14 300 |  |
| **“counting on”** |  |  |
| *It is interesting to note that* |  |  |
|  |  |
| *finding the difference is* |  |
|  |  |
| *reversible. For example, the* |  |  |
| *difference between 5 and 2 is the* |  |  |
| *same as the difference between 2* |  |  |
| *and 5. This is not the case for* |  |  |
| *other subtraction concepts.* |  |  |
|  |  |  |
| *Addition strategies can be* |  |  |
| Finding the difference is efficient when the numbers are close to  each other 9012 –8976 | |  |
|  |
|  |  |  |
|  | |  |
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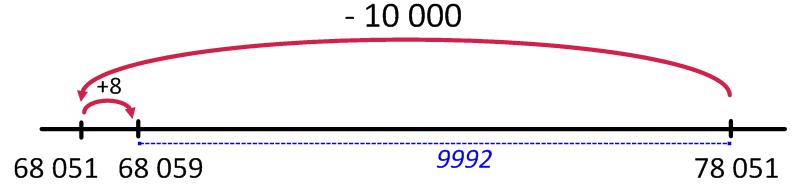


|  |  |  |
| --- | --- | --- |
| **Strategies & Guidance** | **CPA** |  |
|  |  |  |
| **Round and adjust** | **Addition** |  |
| **Addition and subtraction** |  |  |
| **using compensation** |  |  |
| *Pupils should recognise that this* |  |  |
| *strategy is useful when adding* |  |  |
| *and subtracting near multiples* |  |  |
| *of ten. They should apply their* |  |  |
| *knowledge of rounding.* | 54 128 + 9987 = 54 128 + 10 000 –13 = 64128 - 13 |  |
| *It is very easy to be confused* | Pupils should realise that they can adjust first: |  |
| *about how to adjust and so* |  |
|  |  |
| *visual representations and* |  |  |
| *logical reasoning are essential* |  |  |
| *to success with this strategy.* |  |  |

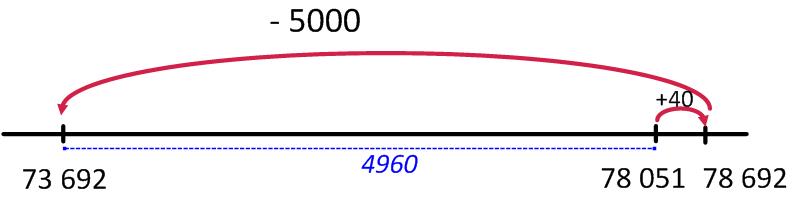


54128 + 9987 = 54128 –13 + 10 000 = 54 115 + 10 000

**Subtraction**



78 051 –9992 = 78 051 –10 000 + 8 = 68 051 + 8 Pupils should realise that they can adjust first:



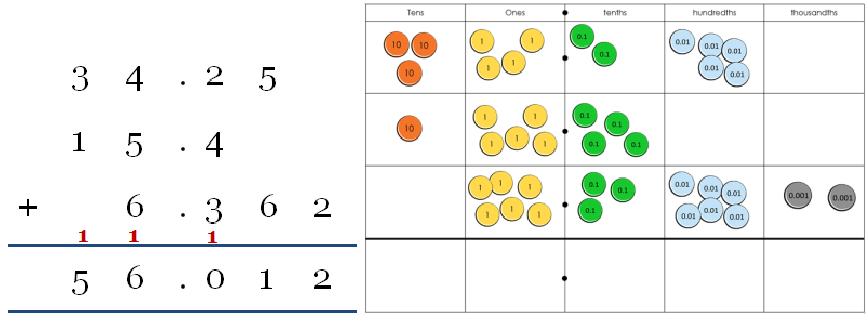
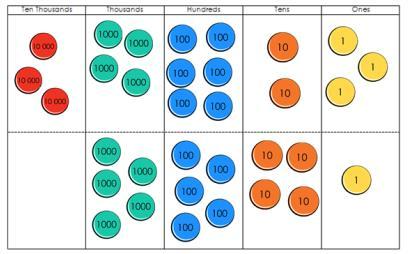
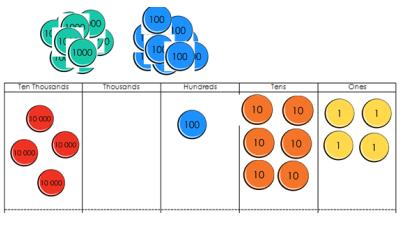
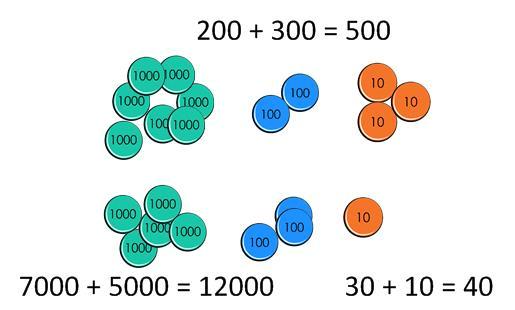
|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | 78 051 –4960 = 78 051 + 40 –5000 = 78 | | | | 692 - 5000 |  |
|  |  |  |  |  |  |  |
| **Near doubles** |  |  |  |  |  |  |
| *Pupils should be able to double* | 160 | + 170 | = double 150 | + 10 + 20 |  |  |
|  |  |  |  |  |  |
| *numbers up to 100 and use this* | 160 | + 170 | = double 160 | + 10 or 160 | + 170 = double 170 - 10 |  |
| *to derive doubles for multiples of* |  |  |  |  |  |  |
| *ten as well as decimal numbers.* | 2.5 + 2.6 = double 2.5 + 0.1 | | | |  |  |
| *These facts can be adjusted to* |  |  |  |  |  |  |
| *calculate near doubles.* |  |  |  |  |  |  |

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| **Strategies & Guidance** |  |  |  |  |  |  |  | **CPA** |  |
|  |  |  |  | | | | |  |  |
| **Partition both numbers and** | 7230 + 5310 = 12 000 + 500 + 40 | | | | | | | |  |
| **combine the parts** |  |  |  |  |  |  |  |  |  |
| *Pupils should be secure with this* |  |  |  |  |  |  |  |  |  |
| *method for numbers up to 10* |  |  |  |  |  |  |  |  |  |
| *000, using place value counters* |  |  |  |  |  |  |  |  |  |
| *or Dienes to show conceptual* |  |  |  |  |  |  |  |  |  |
| *understanding.* |  |  |  |  |  |  |  |  |  |
| *If multiple regroupings are* |  |  |  |  |  |  |  |  |  |
| *required, then pupils should* |  |  |  |  |  |  |  |  |  |
| *consider using the column* |  |  |  |  |  |  |  |  |  |
| *method.* |  |  | Pupils should be aware that the parts can be added in any order. | | | | | |  |
|  |  |  |  | | | | | |  |
| **Written column methods** |  |  | For this method start with the digit of least value because if | | | | | |  |
| **for addition** |  |  | regrouping happens it will affect the digits of greater value. | | | | | |  |
|  |  |  |  |
| *In Year 5, pupils are expected to* |  |  |  |  |  |  |  |  |  |
| *be able to use formal written* |  |  |  |  |  |  |  |  |  |
| *methods to add whole numbers* | 3 | | | 4 | 6 | 2 | 3 |  |  |
| *with more than four digits as* |  |  |
|  |  |  |  |  |  |  |  |  |
| *well as working with numbers* | + | | | 5 | 5 | 4 | 1 |  |  |
| *with up to three decimal places.* |  |  |
|  |  |  |  |  |  |  |  |  |
| *Pupils should think about* |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
| *whether this is the most efficient* |  |  |  |  |  |  |  |  |  |
| *method, considering if mental* |  |  |  |  |  |  |  |  |  |
| *methods would be more* |  |  | Combine the counters in each column and regroup as needed: | | | | | |  |
| *effective.* |  |  |  |
|  |  |  |  |  |  |  |  |  |
| *Continue to use concrete* | 3 | | | 4 | 6 | 2 | 3 |  |  |
| *manipulatives alongside the* |  |  |  |  |  |  |  |  |  |
| *formal method.* | + | | | 5 | 5 | 4 | 1 |  |  |
| *When adding decimal numbers* |  | | 1 | 1 |  |  |  |  |  |
| 4 | | | 0 | 1 | 6 | 4 |  |  |
| *with a different number of* |  |  |
|  |  |  |  |  |  |  |  |  |
| *decimal places, in order to avoid* |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
| *calculation errors, pupils should* |  |  |  |  |  |  |  |  |  |
| *be encouraged to insert zeros so* |  |  | Decimal numbers: | | | |  |  |  |
| *that there is a digit in every* |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
| *row. This is not necessary for* |  |  |  |  |  |  |  |  |  |
| *calculation and these zeros are* |  |  |  |  |  |  |  |  |  |
| *not place holders as the value of* |  |  |  |  |  |  |  |  |  |
| *the other digits is not changed* |  |  |  |  |  |  |  |  |  |
| *by it being placed.* |  |  |  |  |  |  |  |  |  |
| *Exemplification of this method* |  |  |  |  |  |  |  |  |  |
| *and the language to use are best* |  |  |  |  |  |  |  |  |  |
| *understood through viewing the* |  |  |  |  |  |  |  |  |  |
| *tutorial videos found on the* |  |  |  |  |  |  |  |  |  |
| *toolkit.* |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |

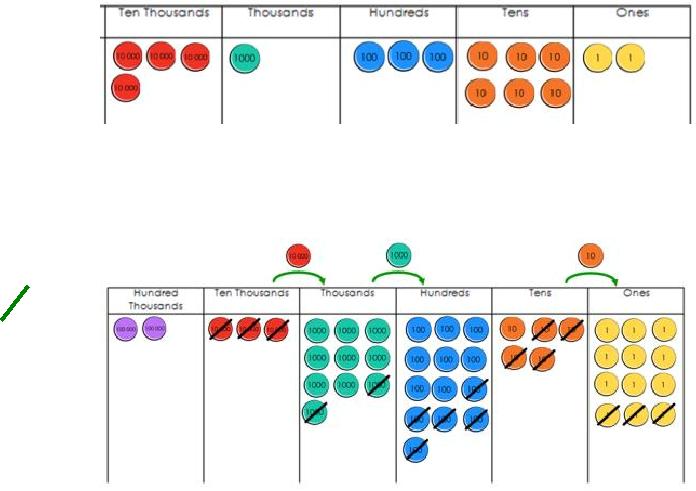


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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Strategies & Guidance** |  | |  |  |  |  |  | **CPA** |  |
|  |  | |  |  |  |  |  |  |  |
| **Written column methods** |  | | 4 | 1 | 3 | 6 | 2 |  |  |
| **for subtraction** |  |  |  |  |
|  | |  |  |  |  |  |  |  |
| *In Year 5, pupils are expected to* |  |  | - 3 | 2 | 2 | 4 | 3 |  |  |
|  | |  |  |  |  |  |  |  |
| *be able to use formal written* |  | |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| *methods to subtract whole* |  | |  |  |  |  |  |  |  |
| *numbers with more than four* |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
| *digits as well as working with* |  | |  |  |  |  |  |  |  |
| *numbers with up to three* |  | | 3 | 1 | 3 | 5 | 1 |  |  |
| *decimal places.* |  |  | 4 | 1 | 6 | 2 |  |  |
|  | |  |  |  |  |  |  |  |
| *Pupils should be given plenty of* |  | | - 3 | 2 | 2 | 4 | 3 |  |  |
| *practice with calculations that* |  | |  |  |  |  |  |  |  |
|  | 9 | 1 | 1 | 9 |  |  |
| *require multiple separate* |  | |  |  |  |
| *instances of regrouping.* |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
| *In Year 3 and 4 they become* |  | | The term regrouping should be the language used. You can use | | | | | |  |
| *more familiar with calculations* |  |  |  |
| the terms ‘exchange’ with subtraction but it needs careful | | | | | | | |  |
| *that require ‘regrouping to* |  |
| *regroup’. Understanding must* | consideration. | | | | | |  |  |  |
| *be secured through the* |  | |  |  |  |  |  |  |  |
| *considered use of manipulatives* |  | | You can regroup 62 as 50 and 12 (5 tens and 12 ones) instead of | | | | | |  |
| *and images, combined with* |  | | 60 and 2 (6 tens and 12 ones). | | | | | |  |
| *careful use of language.* |  | |  |  |  |  |  |  |  |
| *Pupils should think about if this* |  |  | Or you can ‘exchange’ one of the tens for 10 ones resulting in 5 | | | | | |  |
|  | | tens and 12 ones. | | | | | |  |
| *is the most efficient method,* |  |  |  |
|  | |  |  |  |  |  |  |  |
| *considering whether mental* |  | | If you have exchanged, then the number has been regrouped. | | | | | |  |
| *strategies (such as counting on,* |  |  |  |
|  | |  |  |  |  |  |  |  |
| *using known number facts,* |  | |  |  |  |  |  |  |  |
| *compensation etc.) may be* |  | |  |  |  |  |  |  |  |
| *likelier to produce an accurate* |  | |  |  |  |  |  |  |  |
| *solution.* |  | |  |  |  |  |  |  |  |
| *Exemplification of this method* |  | |  |  |  |  |  |  |  |
| *and the language to use are best* |  | |  |  |  |  |  |  |  |
| *understood through viewing the* |  | |  |  |  |  |  |  |  |
| *tutorial videos found* *on the* |  | |  |  |  |  |  |  |  |
| *toolkit.* |  | |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |



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Progression in calculations

Year 5 + Year 6

**National Curriculum objectives linked to multiplication and division**

**These objectives are explicitly covered through the strategies outlined in this document:**

* multiply and divide whole numbers by 10, 100 and 1000
* multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
* multiply and divide numbers mentally drawing upon known facts
* divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
* multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
* divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
* divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
* multiply one-digit numbers with up to two decimal places by whole numbers
* use written division methods in cases where the answer has up to two decimal places

**The following objectives should be planned for lessons where new strategies are being introduced and developed:**

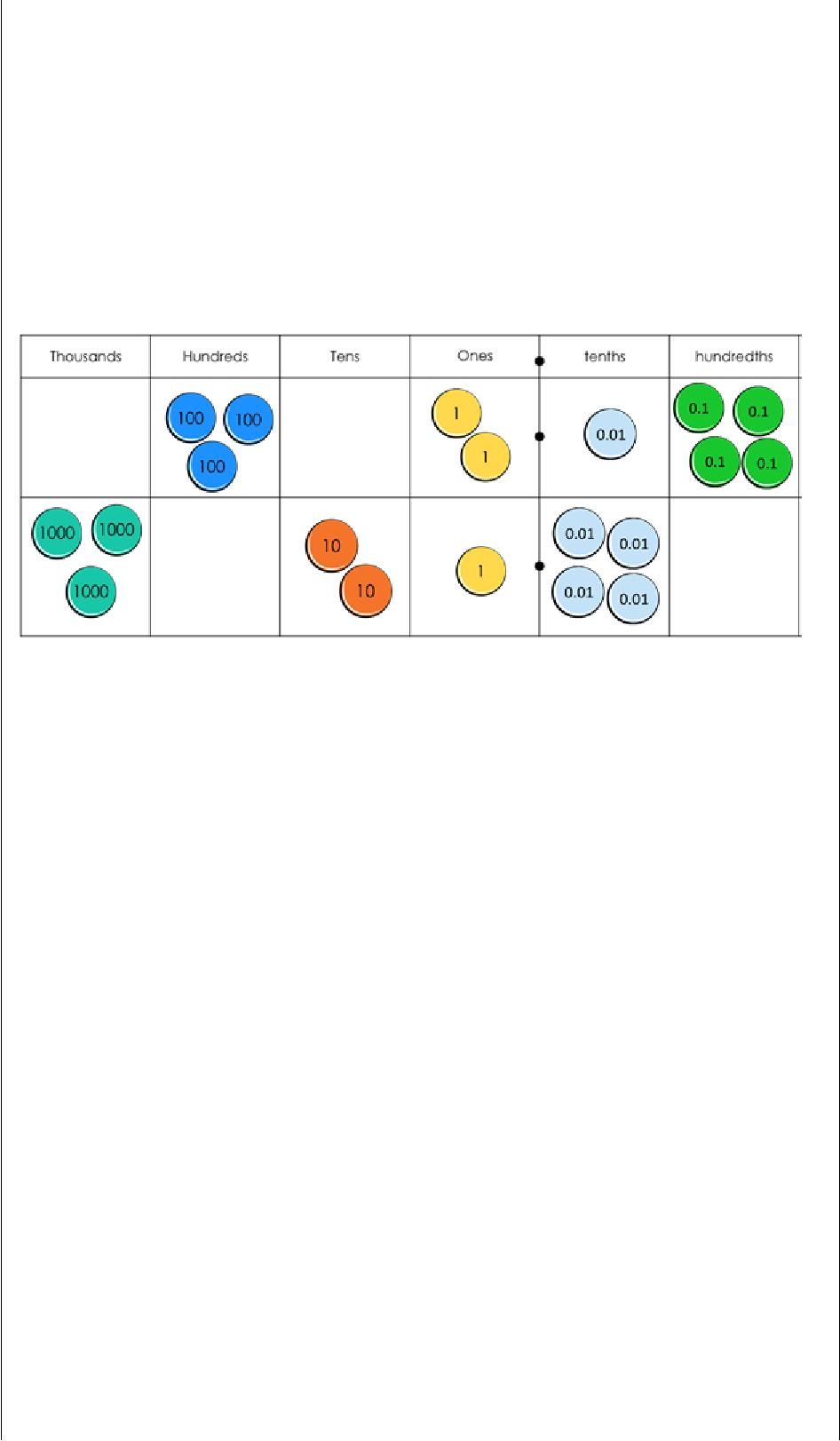
* solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes
* solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
* use their knowledge of the order of operations to carry out calculations involving the four operations
* solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
* solve problems involving addition, subtraction, multiplication and division
* solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts.

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| --- | --- | --- |
|  | **Y5 and Y6 Multiplication** |  |
|  |  |  |
| **Strategies & Guidance** | **CPA** |  |
|  |  |  |
| **Multiply and divide whole** | When you multiply by ten, each part is ten times greater. The ones |  |
| **numbers and those** | become tens, the tens become hundreds, etc. |  |
| **involving decimals by 10,** |  |
|  |  |
| **100 and 1000** | When multiplying whole numbers, a zero holds a place so that each |  |
| *Avoid saying that you “add* | digit has a value that is ten times greater. |  |
|  |  |
| *a zero” when multiplying* | 102.14 x 10 = 1021.4 |  |
| *by ten and instead use the* |  |
|  |  |
| *language of place holder.* |  |  |
| *Use place value counters* |  |  |
| *and charts to visualise and* |  |  |
| *then notice what happens* |  |  |
| *to the digits.* |  |  |

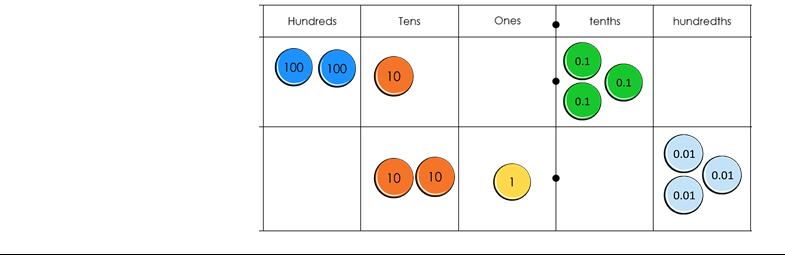


When you divide by ten, each part is ten times smaller. The hundreds become tens and the tens become ones. Each digit is in a place that gives it a value that is ten times smaller.

When dividing multiples of ten, a place holder is no longer needed so that each digit has a value that is ten times smaller.

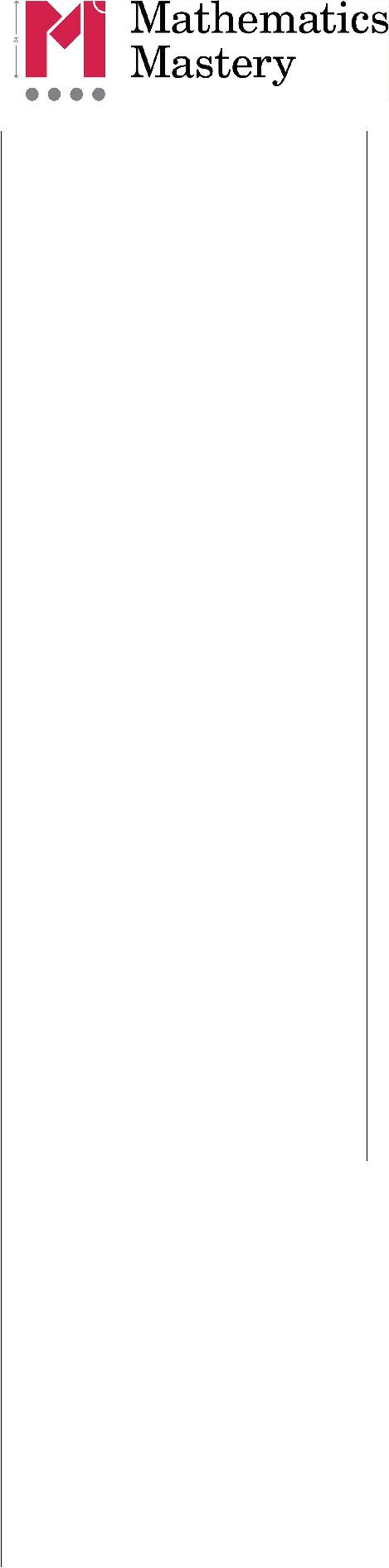
E.g. 210 ÷ 10 = 21

210.3 ÷ 10 = 21.03



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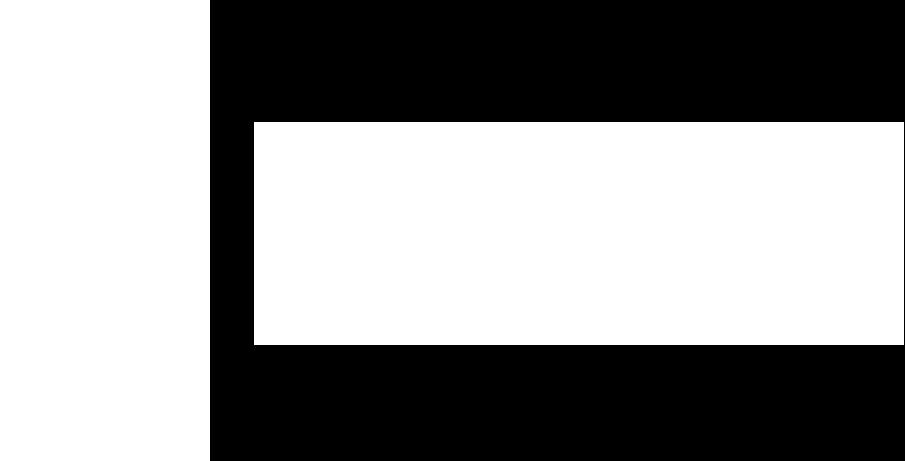
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| **Strategies & Guidance** | **CPA** |
|  |  |
| **Using known facts and** |  |
| **place value to derive** |  |
| **multiplication facts** | Children are encouraged to look for significant figures |

Smile representation from big maths

*Emphasis is placed on understanding the relationship (10 times or 100 times greater) between a known number fact and one to be derived, allowing far*



*larger ‘fact families’ to be derived from a single known number fact.*

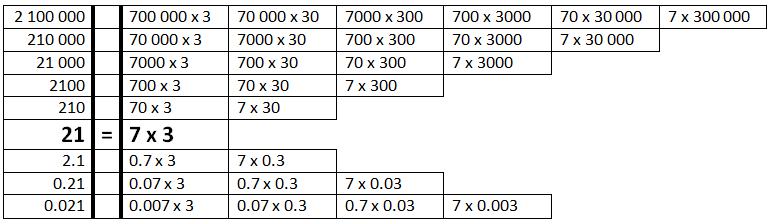
*Knowledge of commutativity is further extended and applied to find a range of related facts.*



*Pupils should work with decimals with up to two decimal places.*

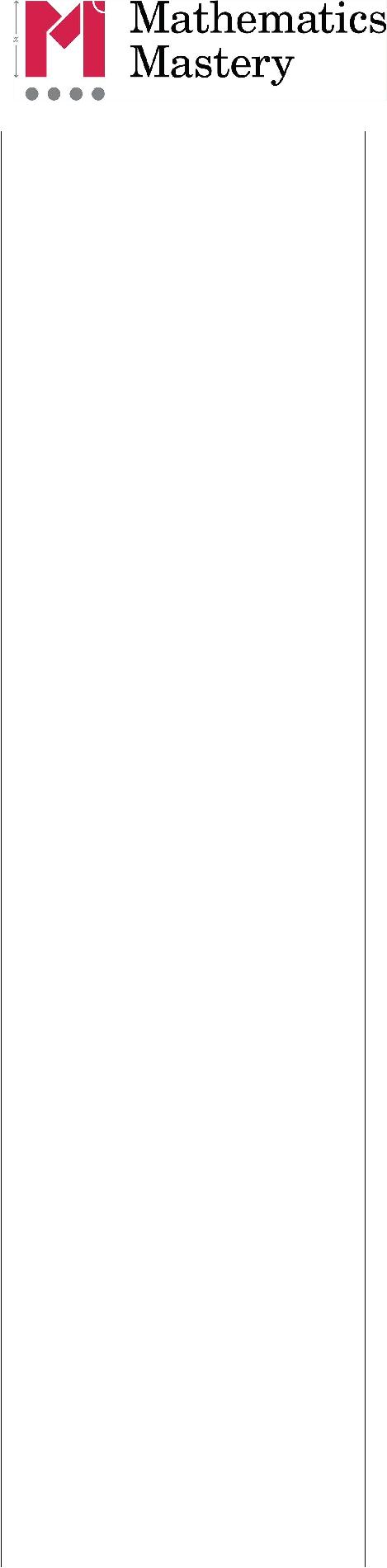
*These derived facts should be used to estimate and check answers to calculations.*

These are the multiplication facts pupils should be able to derive from a known fact

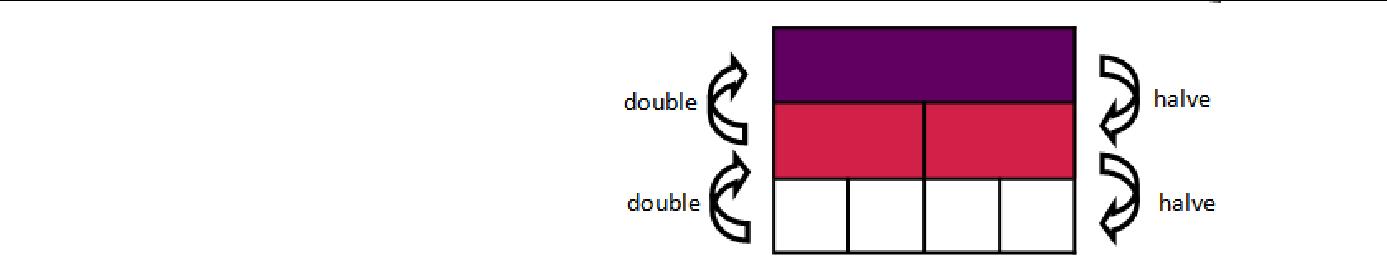


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| **Strategies & Guidance** | **CPA** |



**Doubling and halving**

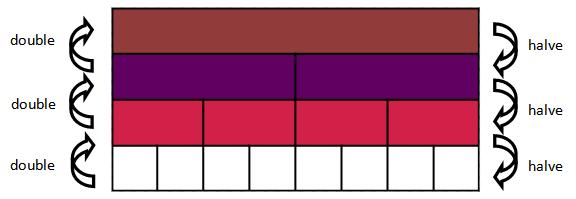
*Pupils should experience doubling and halving larger and smaller numbers as they expand their understanding of the number system.*

*Doubling and halving can* **Multiply by 4** by doubling and doubling again

*then be used in larger*

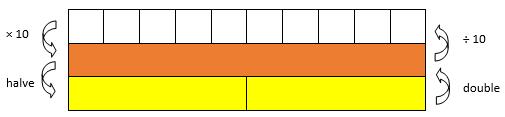
*calculations.* e.g. 16 × 4 = 32 × 2 = 64

**Divide by 4** by halving and halving againe.g. 104 ÷ 4 = 52 ÷ 2 = 26



**Multiply by 8** by doubling three timese.g. 12 × 8 = 24 × 4 = 48 × 2 = 96

**Divide by 8** by halving three timese.g. 104 ÷ 8 = 52 ÷ 4 = 26 ÷ 2 = 13



**Multiply by 5** by multiplying by 10 then halving,e.g. 18 × 5 = 180 ÷ 2 = 90.

**Divide by 5** by dividing by 10 and doubling,e.g. 460 ÷ 5 = double 46 = 92

**Links to fraction**

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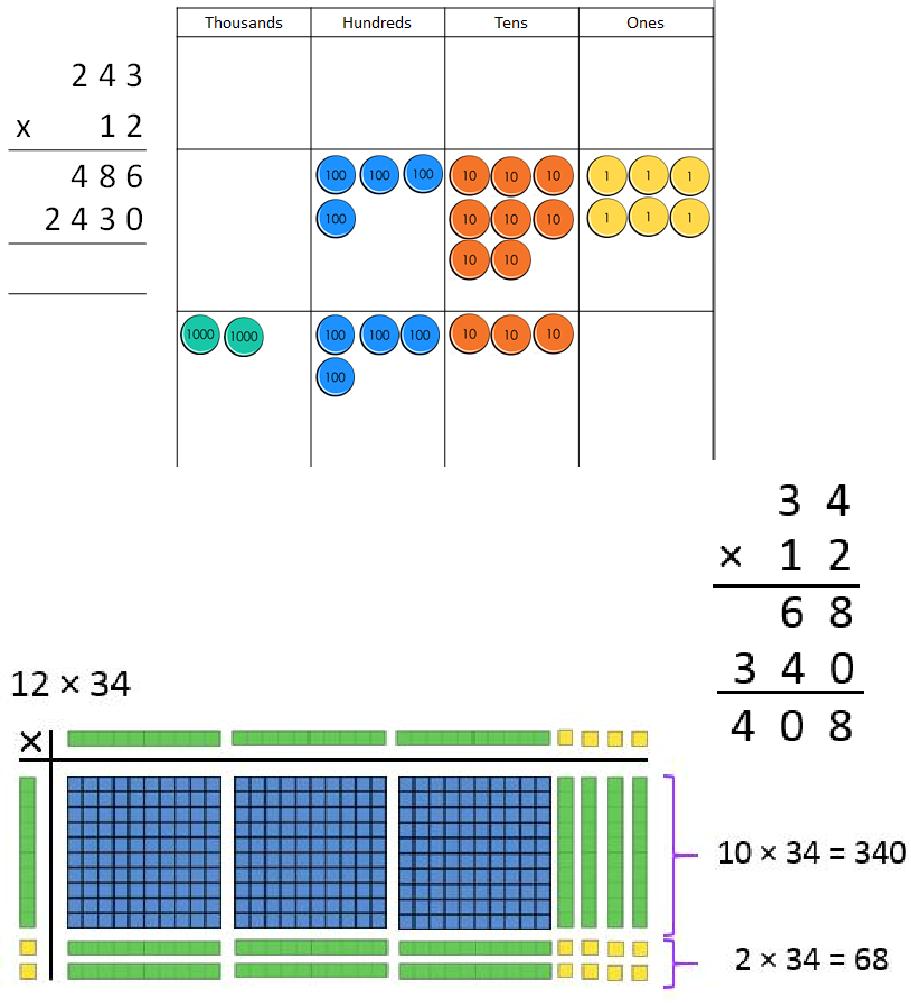
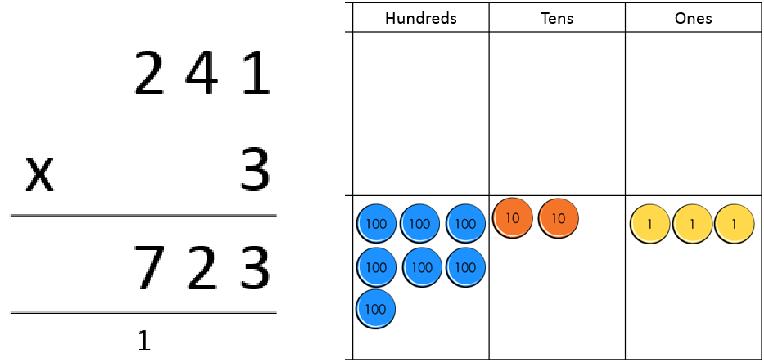


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| --- | --- | --- | --- |
| **Strategies & Guidance** | **CPA** |  |  |
|  |  |  |  |
| **Multiply by partitioning** | 8 x 14 = 8 x 10 + 8 x 4 - Year 6 |  |  |
| **one number and** |  |  |  |
| **multiplying each part** |  |  |  |
| **Distributive law** |  |  |  |
| **a x (b + c) = a x b + a x c** | Cuisenaire rods to build arrays | Represent with area model |  |
| Build on pupils’ understanding | Year 5 area models |  |  |
| of arrays of counters to |  |  |  |
| represent multiplication to |  | |  |
| see that area models can be a |  |  |  |
| useful representation: |  |  |  |
|  |  | |  |
| **Using knowledge of** | Calculate 6 x 24 by using factor pairs of 24 | |  |
| **factors** |  |  |  |
| *In Year 5 pupils are expected* |  |  |  |
|  |  |  |
| *to be able to identify factor* |  |
| Children are taught to recognise factors in pairs and write these down systematically to ensure they have not missed any.  Factor bugs are introduced in year 5 as a starting point however once understood children write down underneath the number.  e.g.  factors of 24  1 x 24  2 x 12  3 x 8  4 x 6 |  |  |
| *pairs and this knowledge can* |  |  |  |
| *be used to calculate.* |  |  |  |
| *Pupils will be using the* |  |  |  |
| *commutative and associative* |  |  |  |
| *laws of multiplication.* |  |  |
|  |  |  |
|  |  |
| **Commutative law** |  |  |  |
| **a x b = b x a** |  |  |  |
| **Associative law** |  |  |  |
|  |  |  |
| **a x b x c = (a x b) x c** |  |  |  |
| **= a x (b x c)** |  |  |  |
| *They should explore and* |  |  |  |
| *compare the different options* |  |  |  |
| *and choose the most efficient* |  |  |  |
| *order to complete* |  |  |  |
| *calculations.* |  |  |  |
|  |  |  |  |

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| --- | --- | --- | --- | --- | --- | --- | --- |
| **Strategies & Guidance** | | | **CPA** | | | |  |
|  |  |  |  |  |  |  |  |
| **Formal written method of** | | |  |  |  |  |  |
| **short multiplication** | | |  |  |  |  |  |
| *Conceptual understanding is* | | |  |  |  |  |  |
| *supported by the use of place* | | |  |  |  |  |  |
| *value counters, both during* | | |  |  |  |  |  |
| *teacher demonstrations and* | | |  |  |  |  |  |
| *during their own practice.* | | |  |  |  |  |  |
| *Exemplification of this* | | |  |  |  |  |  |
| *method and the language to* | | |  |  |  |  |  |
| *use are best understood* | | |  |  |  |  |  |
| *through viewing the tutorial* | | |  |  |  |  |  |
| *videos found on the* | | |  |  |  |  |  |
| *toolkit.* | | |  |  |  |  |  |
|  | | |  |  |  |  |  |
| **Multiplying by a 2-digit** | | |  |  |  |  |  |
| **number** | | |  |  |  |  |  |
| **Formal written method of** | | |  |  |  |  |  |
| **long multiplication** | | |  |  |  |  |  |
|  |  |  |  |  | |  |  |
|  |  |  |  | 243 x 2 |  |  |  |
| *In* | *Year 6* | *pupils are extended* |  |
|  |  |  |  |  |
| *from multiplication by a 1-* | | |  |  |  |  |  |
| *digit number to multiplication* | | |  |  |  |  |  |
|  | 243 x 10 | |  |  |
| *by a 2-digit number.* | | |  |  |  |
| *Extend the place value chart* | | |  |  |  |  |  |
|  |  |  |  |  |
| *model used in Year 4, using* | | |  |  |  |  |  |
| *an additional row on the* | | |  |  |  |  |  |
| *place value chart.* | | |  |  |  |  |  |
| *Extend understanding of the* | | |  |  |  |  |  |
| *distributive law to develop* | | |  |  |  |  |  |
| *conceptual understanding of* | | |  |  |  |  |  |
| *the two rows of the formal* | | |  |  |  |  |  |
| *written method.* | | |  |  |  |  |  |
| *Dienes blocks can be used to* | | |  |  |  |  |  |
| *construct area models to* | | |  |  |  |  |  |
| *represent this.*  ***Year 5*** | | |  |  |  |  |  |
|  |  |  |  |  |  |  |  |

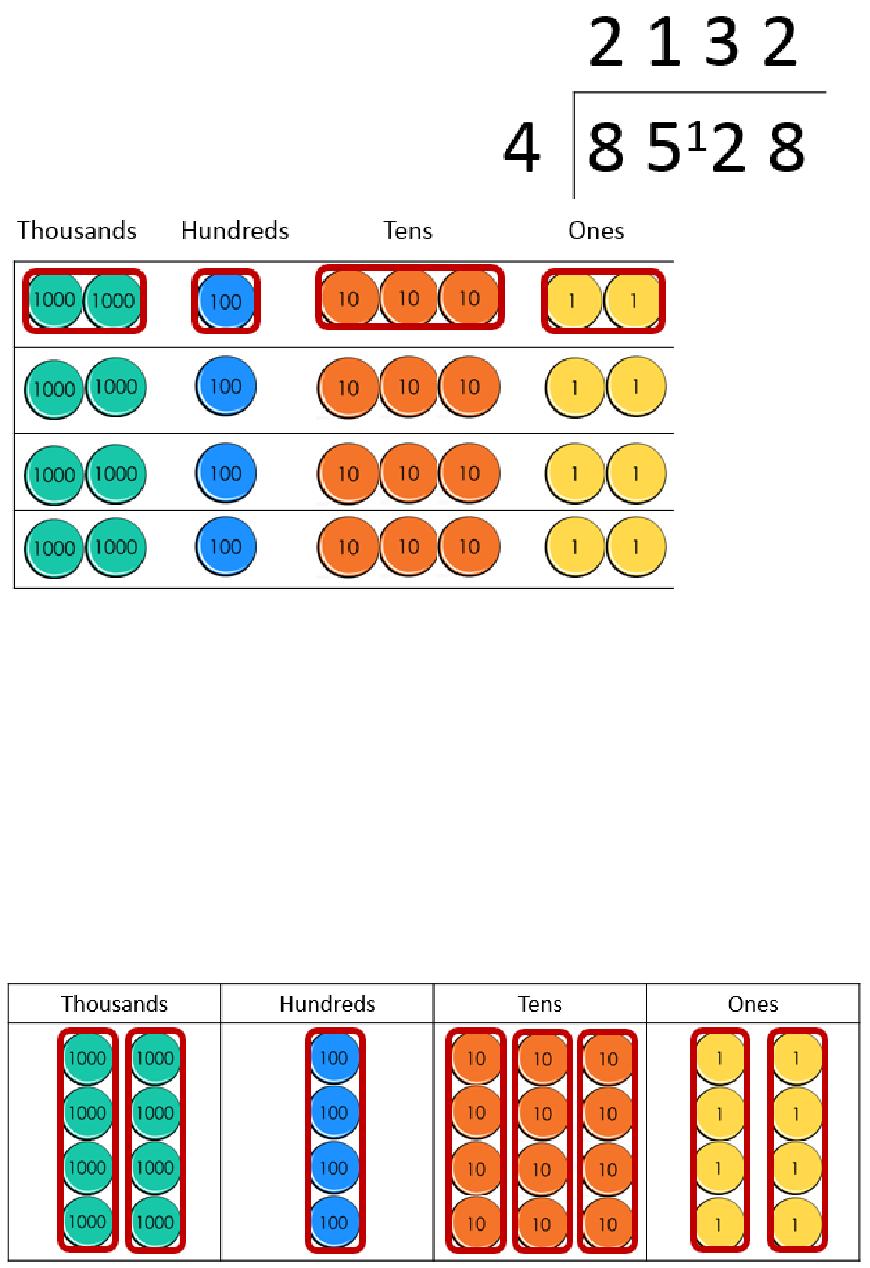


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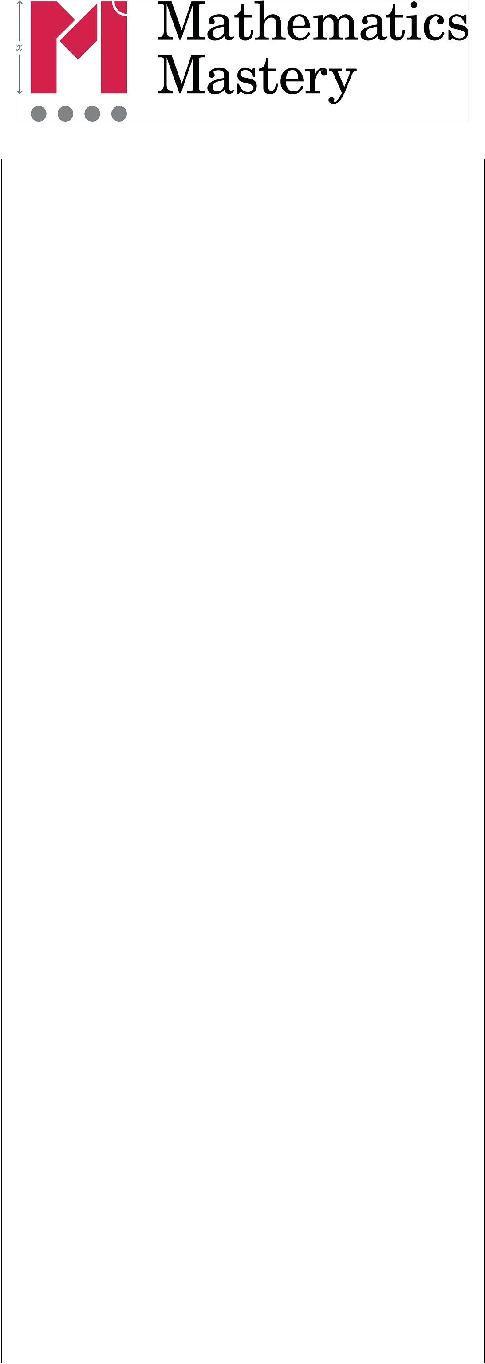


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| **Strategies & Guidance** | **CPA** |  |
|  |  |  |
| **Short division** | 8528 ÷ 4 |  |
| **Dividing a 4-digit numbers** |  |  |
| **by 1-digit numbers** |  |  |
| *The thought process of the* | **Sharing** |  |
|  |  |
| *traditional algorithm is as* |  |  |
| *follows:* |  |  |
| *How many 4s in 8? 2* |  |  |
| *How many 4s in 5? 1 with 1* |  |  |
| *remaining so regroup.* |  |  |
| *How many 4s in 12? 3* |  |  |
| *How many 4s in 8? 2* |  |  |
| *Warning: If you simply apply* |  |  |
| *place value knowledge to each* |  |  |
| *step, the thinking goes wrong if* | 8 thousands shared into 4 equal groups |  |
| *you have to regroup.* |  |
|  |  |
|  | 5 hundreds shared into 4 equal groups |  |
| *How many 4s in 8000? 2000* | Regroup 1 hundred for 10 tens |  |
| *How many 4s in 500?* |  |
| 12 tens shared into 4 equal groups |  |
| *100 with 1 remaining (illogical)* |  |
| 8 ones shared into 4 equal groups. |  |
| *The answer would be 125.* |  |
|  |  |
| *Sharing the dividend builds* | **Grouping** |  |
| *conceptual understanding* |  |  |
| *however doesn’t scaffold the* |  |  |
| *“thinking” of the algorithm* |  |  |
| *Using place value counters and* |  |  |
| *finding groups of the divisor for* |  |  |
| *each power of ten will build* |  |  |
| *conceptual understanding of the* |  |  |
| *short division algorithm.* |  |  |
| *Area models are also useful* | How many groups of 4 thousands in 8 thousands? |  |
|  |  |
| *representations, as seen with* | How many groups of 4 hundreds in 5 hundreds? |  |
| *other strategies and exemplified* | Regroup 1 hundred for 10 tens. |  |
| *for long division.* | How many groups of 4 tens in 12 tens? |  |
|  |  |
| *Exemplification of this method* | How many groups of 4 ones in 8 ones? |  |
|  |  |
| *and the language to use are best* |  |  |
| *understood through viewing the* |  |  |
| *tutorial videos found* *on the* |  |  |
| *toolkit.* |  |  |
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| **Strategies & Guidance** | **CPA** |
|  |  |
| **Long division taught using the**  **Formal method of division**  **Bus stop**  **Children are encouraged to**  **write down multiples to help** |  |

**Dividing a 4-digit number by a 2-digit number**

*If dividing by a number outside of their known facts, pupils should start by recording some multiples of that number to scaffold.*



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